179. On Countably R-closed Spaces. II

By Masao SAKAI

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A topological space S is called *countably R-closed*, if for any family $\{G_n\}_{n=1}^{\infty}$ of nonvoid open sets such that $G_n \supset \bar{G}_{n+1}$ for every n, we have $\bigcap_{n=1}^{\infty} G_n \neq \phi$. Z. Frolik [1] and the present author [2] gave characterizations of countably R-closed spaces.

In any topological space S, if a family Φ, composed of subsets of S, has a point x such that each neighbourhood of x meets infinitely many members of Φ, we say that *Φ cluster to x* and that the point x is *a cluster point of Φ*. S. Kasahara [3] proved the following:

Proposition. *In any regular T_1-space S, the following conditions are equivalent:*

(i) *Every family of pairwise disjoint open sets has at least one cluster point.*

(ii) *Every star-finite open covering of S has a finite subcovering.*

(iii) *Every star-finite open covering of S has finite subfamily whose union is dense in S.*

We shall give another characterization of countably R-closed regular spaces, using the method of S. Kasahara [3].

In any topological space S, a family Φ composed of subsets of S is called *locally finite* if every point x has a neighbourhood $U(x)$ which meets only finite members of Φ, and Φ is called *star-finite* if every member of Φ meets only finite members of Φ. A subset E is called *regularly closed* if E is the closure of an open set of S. A covering of S composed of regularly closed sets is called *a regularly closed covering of S*.

Theorem. *In any regular space S, the following conditions are equivalent:*

(1) *S is countably R-closed.*

(2) *Every family of pairwise disjoint regularly closed sets has at least one cluster point.*

(3) *Every family of pairwise disjoint open sets has at least one cluster point.*

We shall prove that (1)→(2)→(1) and (2)→(3)→(2). In stead of (1), we shall use (4) and (5) of the following Lemma 1.

Lemma 1. *In any topological space S, the following conditions are equivalent:*

(1) *S is countably R-closed.*

(4) *Every locally finite, star-finite, countable, regularly closed*