# 178. On Countably R-closed Spaces. I

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A topological space $S$ is called *countably R-closed*, if for any family $\{G_n\}_{n=1}^{\infty}$ of nonvoid open sets such that $G_n \supset \bar{G}_{n+1}$ for every $n$, we have $\bigcap_{n=1}^{\infty} G_n \neq \phi$. Z. Frolik [1] proved the following:

**Proposition.** *In any topological space $S$, the following properties are equivalent:*

( i ) *$S$ is countably $R$ closed.*

(ii) *Every star-finite open covering of $S$ has a finite subfamily whose union is dense in $S$.*

(iii) *Every star-finite open covering of $S$ has a finite subcovering.*

(iv) *Every star-finite open covering of $S$ is a finite covering.*

We shall give other characterizations of countably $R$-closed spaces. In a topological space $S$, a family $\Phi$ composed of subsets of $S$ is called *locally finite* (*discrete*) if every point $x$ has a neighbourhood $U(x)$ which meets only finite members (at most only one member) of $\Phi$, and $\Phi$ is called *star-finite* if every member of $\Phi$ meets only finite members of $\Phi$. A subset $E$ is called *regularly closed* if $E$ is the closure of an open set of $S$. A covering of $S$ composed of regularly closed sets is called *a regularly closed covering of $S$.*

**Theorem.** *In any topological space $S$, the following conditions are equivalent:*

(1) *$S$ is countably $R$-closed.*

(2) *Every locally finite, star-finite, countable, regularly closed covering of $S$ has a finite subcovering.*

(3) *Every locally finite, star-finite, countable, regularly closed covering of $S$ is a finite covering.*

(4) *Every locally finite, star-finite, regularly closed covering of $S$ is a finite covering.*

(5) *Every star-finite open covering of $S$ is a finite covering.*

We shall prove that (1)$\to$(2)$\to$(3)$\to$(1) and (3)$\to$(4)$\to$(5)$\to$(4)$\to$(3).

**Lemma 1.** *In a topological space $S$, let $\{\bar{O}_n\}_{n=1}^{\infty}$ be a locally finite, countable, regularly closed covering of $S$. Then $F_n = \bigcup_{k=n+1}^{\infty} \bar{O}_k$ is closed, $G_n = S - \bigcup_{l=1}^{n} \bar{O}_l$ is open, and $F_n \supset G_n$ for every $n$.*

**Lemma 2.** *Let $\{\bar{O}_n\}_{n=1}^{\infty}$ be a locally finite, star-finite, countable, regularly closed covering of $S$. For every $n$, there is $m$ ($\geqq n+1$) such that $F_m \subset G_n$.*