232. A Note on Spaces with a Uniform Base

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In this note, we shall consider some properties in connection with the spaces with a uniform base. The notion of a uniform base was introduced by Aleksandrov [1]. A collection $\mathcal{B}$ of open sets in a space X is a *uniform base* if for each $x \in X$, any infinite subset of $\mathcal{B}$, each member of which contains x, is a local base at x. In [1] it is proved that a space X has a uniform base if and only if X has a development consisting of point-finite open coverings of X. Arhangel'skii [2] obtained that a T_1-space X has a uniform base if and only if X is an open, compact (continuous) image of some metric space. From these facts, it is known that a T_1-space X has a uniform base if and only if X is a metacompact (=point-paracompact), developable space. Also it is clear that a space with a uniform base has a σ-point-finite base. However, Example 6.4 of [8] shows that the converse of this result is not true in general (cf. [3]). Spaces are assumed to be T_1.

1. Characterizations of spaces with a uniform base. Recently the author has been informed that F. Siwiec has proved the following: *A T_1-space X has a uniform base if and only if X has a σ-point-finite base and each closed set of X is a G_δ-set,* and that he has asked to prove directly that the above condition for X implies X being an open, compact image of a metric space. We shall prove this in the proof of the following Theorem 1 which contains other characterizations of spaces with a uniform base.

Theorem 1. *For a T_1-space X, the following conditions are equivalent:*

1) *X is an open, compact image of a metric space,*
2) *X is a metacompact Σ-space with a point-countable base,*
3) *X is a $w\Delta$-space with a σ-point-finite base,*
4) *X is a $\Sigma^{\#}$-space with a σ-point-finite base.*

Proof. 1)$\to$2). It is easy to show that X has a development $\{\mathcal{V}_i : i=1, 2, \cdots\}$ consisting of point-finite open coverings of X. Therefore X is metacompact and developable. Since X is a developable space, X has a σ-locally finite closed net and hence is a Σ-space.

2)$\to$3). Since X is a T_1 Σ-space with a point-countable base, X is a developable space by [16, Corollary 1.3] and hence X is a $w\Delta$-space. Since X is a metacompact developable space, X has a σ-point-finite base.