231. A Remark on the Boundary Behavior of $(Q)L_1$-Principal Functions

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(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 13, 1971)

Let R be an open Riemann surface and Q be the canonical partition of the ideal boundary of R. The problem characterizing $(Q)L_1$-principal functions by the boundary behavior under compactifications has been investigated by several authors (Sario-Oikawa [9]). The class of $(Q)L_1$-principal functions has been shown to be identical with the class of single-valued canonical potentials introduced by Kusunoki [5] (Watanabe [10]). As a necessary condition, the fact that a $(Q)L_1$-principal function can be extended almost everywhere (or quasi-everywhere) continuously on some compactifications so that the extension is a.e. (q.e.) constant on each component of the ideal boundary has been proved by some authors in different ways (Ikegami [3], Kusunoki [6] and Watanabe [10]).

Then, the question arises whether, conversely, this boundary property would be sufficient for a function to be a $(Q)L_1$-principal function.

Watanabe [10] showed a sufficient condition in the following particular form. Suppose that a real-valued harmonic function f with a finite number of singularities is Dirichlet integrable in a boundary neighborhood U and $\int_\gamma {}^*df=0$ for any dividing cycle γ in U, and is almost everywhere constant on each boundary component of a compactification R^*. The R^* may be one of Martin, Royden, Wiener, Kuramochi or a $\mathfrak{Q}$-compactification denoting by $\mathfrak{Q}$ a sublattice of HP which contains constant. If the set of constant values taken by f on boundary components is isolated except the supremum and infimum, then f is a $(Q)L_1$-principal function.

On the other hand, if R is of finite genus, any harmonic function in a boundary neighborhood whose conjugate is semi-exact has a limit at a weak boundary component. Therefore, if a Riemann surface, whose all boundary components are weak, is not of class O_{KD}, there exist functions which are not $(Q)L_1$-principal functions but have limits at any boundary component (Watanabe [10]). However, these functions do not seem to be good enough as counter examples, because the condition 'having limits at weak boundary components' may not be expected to be any restriction.