# 226. *A Note on Nuclear Operators in Hilbert Space*

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**1. Introduction.** Let $\mathfrak{H}$ be a separable Hilbert space and $T$ a (bounded linear) operator on $\mathfrak{H}$. Then $T$ is said to be *nuclear* (or of *trace class*) if there is an orthonormal basis $(e_n)$ in $\mathfrak{H}$ such that

$$(1) \qquad \sum_{n=1}^{\infty} (|T|\, e_n \,|\, e_n) < +\infty,$$

where $|T|$ is the absolute of $T$. Let $\mathfrak{T}$ be the set of all nuclear operators. Then $\mathfrak{T}$ is an algebraical ideal of $\mathfrak{B}(\mathfrak{H})$ which is the algebra of all operators on $\mathfrak{H}$. If $T \in \mathfrak{T}$, then there are orthonormal sets $(e_n)$ and $(f_n)$ such that

$$(2) \qquad T = \sum_{n=1}^{\infty} a_n e_n \otimes f_n,$$

where $(a_n)$ is a sequence of positive numbers such as

$$(3) \qquad \sum_{n=1}^{\infty} a_n < +\infty$$

and

$$(4) \qquad (e \otimes f)g = (g \,|\, f)e,$$

in the notation of Schatten [2].

According to [1], K. Maurin conjectured that an operator $T$ in nuclear if and only if $T$ satisfies

$$(5) \qquad \sum_{n=1}^{\infty} \|Te_n\| < +\infty$$

for any orthonormal set $(e_n)$. The conjecture is recently disproved by J. R. Holub [1]:

**Theorem 1.** *There is a nuclear operator $T$ which does not satisfy* (5) *for an orthonormal basis $(e_n)$.*

Moreover, Holub [1] proves by virtue of a result of J. Lindenstrauss and A. Pelczynski on absolutely summing operators.

**Theorem 2.** *An operator $T$ is nuclear if and only if there is an orthonormal basis $(e_n)$ which satisfies* (5).

In the present note, we shall give simplified proofs of the above theorems in §§ 2–3. Incidentally, we shall characterize operators which satisfy Maurin's conjecture in § 4.

**2. Proof of Theorem 2.** It seems to us that Theorem 2 is already known, e.g. [3, Ex. 3–B, No. 30, p. 69]. However, for the sake of

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