224. *Results Related to Closed Images of M-Spaces.* I

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(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 12, 1971)

1. Introduction. In 1969, J. Nagata began a discussion of characterizations of images of M-spaces under various continuous maps (see, for instance [4] and [5]). Other images of M-spaces have been characterized by Wicke [7], Chiba [1] and by Rishel [6]. One such characterization which has not yet been carried out is that of closed images of M-spaces. It is the purpose of this paper to demonstrate that characterization.

In this paper, all maps are continuous and onto; the symbol "N" will refer to the natural numbers. All spaces will be considered to be T_1-spaces.

2. Preliminaries about covers.

Definition 2.1. A system $\{F_\alpha : \alpha \in \Omega\}$ of closed sets from a space X is said to be *hereditarily closure preserving* if and only if: for any system $\{M_\alpha : \alpha \in \Omega\}$ of closed sets in X such that $M_\alpha \subset F_\alpha$ for every $\alpha \in \Omega$, $\bigcup\{M_\alpha : \alpha \in \Omega\} = \mathrm{Cl}\,[\bigcup\{M_\alpha : \alpha \in \Omega\}]$.

Definition 2.2. A family $\{B_n : n \in N\}$ of sets in a space X is said to form a *q-sequence at* $x \in X$ if and only if:

(a) $x \in B_n$ for every $n \in N$,

(b) for every point-sequence $\{x_n\}$ such that $x_n \in B_n$ for every $n \in N$, $\{x_n\}$ clusters.

Definition 2.3. A sequence of closed covers $\{\mathcal{A}_n\}$ of a space X is said to be *almost q-refining* if and only if for any point $x \in X$, any system of sets $\{B_n\}$, such that $B_n \in \mathcal{A}_n$ for all $n \in N$ and $x \in B_n$ for all $n \in N$, is either hereditarily closure preserving or else forms a q-sequence at x.

Morita [3] originally defined M-spaces.

Definition 2.4. A space X is said to be an *M-space* if and only if there exists a normal sequence of open covers $\{\mathcal{U}_1, \mathcal{U}_2, \cdots\}$ of X satisfying

(1) every point-sequence of the form $\{x_n\}$, where $x_n \in \mathrm{St}(x, \mathcal{U}_n)$ for all n and for fixed $x \in X$, has a cluster point.

Definition 2.5 (Nagata [5]). A space Y is *quasi-k* if and only if, given $F \subset Y$, F is closed whenever $F \cap K$ is relatively closed in K for every

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