

222. Euclidean Space Bundles and Disk Bundles

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0. Introduction.

A useful property of the orthogonal group $O(n)$ is that it leaves invariant the unit disk D^n and unit sphere S^{n-1} of \mathbf{R}^n . Consequently one may pass freely from \mathbf{R}^n -bundles to D^n - and S^{n-1} -bundles in the case where the structure group is $O(n)$. This convenient coincidence does not occur in the topological category. W. Browder [2] showed that some \mathbf{R}^n -bundles do not contain any D^n -subbundles.

In this paper we shall study on the relationship between \mathbf{R}^n -bundles and D^n -bundles.

The main result of this paper is the following

Theorem 1. *Let K be a locally finite simplicial complex of dimension k , and $k < n - 3$ and $n \geq 6$. Then the set of all equivalence classes of D^n -bundles over K is canonically in one-to-one correspondence with the set of all equivalence classes of \mathbf{R}^n -bundles over K .*

In § 1 we prepare on notations and terminologies used later. In § 2 we shall show the stability theorem of the homotopy groups $\pi_k(\mathcal{G}_0(n))$. Here we use the recent result of R. Kirby and L. Siebenmann [4]. In § 3 we shall prove the theorem 1.

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1. Notations and terminologies.

Let $\mathcal{H}_0(n)$ be the space of all homeomorphisms of the Euclidean n -space \mathbf{R}^n onto itself preserving the origin 0 with compact-open topology. Then $\mathcal{H}_0(n)$ forms a topological group with the composition of maps (cf. Kister [5]).

By an \mathbf{R}^n -bundle we shall mean a fibre bundle whose fibre is the Euclidean n -space \mathbf{R}^n and structure group $\mathcal{H}_0(n)$.

Let $B_{\mathcal{H}_0(n)}$ be the classifying space for the topological group $\mathcal{H}_0(n)$. Its existence is assured by J. Milnor [6]. Then for a finite complex K , the set $[K, B_{\mathcal{H}_0(n)}]$ of all homotopy classes of continuous maps of K into $B_{\mathcal{H}_0(n)}$ is in one-to-one correspondence with the set of all equivalence classes of \mathbf{R}^n -bundles over K .

On the other hand, we shall denote by TOP_n the *css*-group of all isomorphism-germs of trivial microbundles over simplexes (for the precise definition, see [1]; where we write H_n for TOP_n). Let B_{TOP_n} be