# 219. On Ergodic and Abelian Automorphism Groups of von Neumann Algebras

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Recently, in [5] Tam proved that any ergodic and abelian automorphism group of an abelian von Neumann algebra is freely acting.

In this paper, we shall give a generalization of Tam's theorem, using the notion of the generalized free action due to Kallman [3]. And we shall generalize Kallman's theorem that all the powers of an ergodic automorphism of a $\mathrm{II}_1$-factor are outer [3].

**1.** Let $\mathcal{A}$ be a von Neumann algebra acting on a Hilbert space $\mathfrak{H}$. In this paper we shall write briefly a *-automorphism of $\mathcal{A}$ as an automorphism of $\mathcal{A}$.

**Definition A.** Let $G$ be a group of automorphisms of a von Neumann algebra $\mathcal{A}$. Then $G$ is called to be *ergodic* on $\mathcal{A}$ if the only $A$ in $\mathcal{A}$ which satisfies

$$(*) \qquad g(A)=A \qquad \text{(for all } g\in G)$$

is scalar. An automorphism $g$ on $\mathcal{A}$ is called to be *ergodic* on $\mathcal{A}$ if the only $A$ in $\mathcal{A}$ which satisfies the condition (*) is scalar.

Kallman [3] has generalized the von Neumann free action for an abelian von Neumann algebra as follows:

**Definition B (Kallman).** An automorphism $g$ on a von Neumann algebra $\mathcal{A}$ is called to be *freely acting* on $\mathcal{A}$ if the only $A$ in $\mathcal{A}$ which satisfies

$$(**) \qquad AB=g(B)A \qquad \text{for all } B\in\mathcal{A}$$

is $A=0$.

The condition (**) in the Definition B is used by Nakamura and Takeda and plays an important role in the theory of the crossed product [4].

Under the Definition B, Kallman showed that any automorphism of a von Neumann algebra is decomposed into freely acting part and inner part. Using this theorem, we have the following:

**Lemma 1.** *Let $\mathcal{A}$ be a von Neumann algebra, $G$ an ergodic group of automorphisms of $\mathcal{A}$ and $\alpha$ an automorphism of $\mathcal{A}$ such that*

$$\alpha g=g\alpha \qquad \textit{for every } g\in G.$$

*Then the automorphism $\alpha$ is freely acting or inner.*

**Proof.** By the Kallman theorem, there exist a central projection $P$ and a unitary operator $U$ in $\mathcal{A}$ such that