216. A Remark on Semi-groups of Local Lipschitzians in Banach Space

By Yoshio KONISHI
Department of Mathematics, University of Tokyo

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1. Introduction. Let X be a Banach space with the norm denoted by $\|\cdot\|$ and let C be a subset of X. A one-parameter family $\{T_t; 0\leq t<+\infty\}$ of (nonlinear) operators of C into itself is called a *semi-group* on C if it satisfies the following conditions:

(i) $T_0=I|_C$ (the identity mapping restricted to C) and $T_{t+s}=T_tT_s$ for $t, s\geq 0$;

(ii) For each fixed $x\in C$, T_tx is strongly continuous in $t\geq 0$.

A (possibly) multiple-valued[1)] operator A (with the domain $D(A)$ and the range $R(A)$) in X is said to be a *D-operator* (in the terminology of Chambers and Oharu [1]) if it satisfies the following condition:

(D) There exists a non-negative function $\omega=\omega(r)$ on $(0, +\infty)$ such that $A|_{B_r}-\omega(r)I$ is dissipative for each $r>0$; where $B_r=\{x\in X; \|x\|\leq r\}$.

The purpose of this paper is to give a sufficient condition in order that a D-operator A in X generate a semi-group on $\overline{D(A)}$ and show some examples. The condition is a modified version of that in [1].

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2. Generation of semi-groups. Our theorem reads:

Theorem 1. *We assume that A is a D-operator in X and that there exists a positive function $\rho=\rho(r, T)$ on $(0, +\infty)\times(0, +\infty)$ satisfying the following condition (S_n) for each sufficiently large integer $n>2T\omega(\rho(r, T))$:*

(S_n) *The system of equations:*

$$x_\lambda^{(1)}-\lambda Ax_\lambda^{(1)}\ni x,\quad x_\lambda^{(2)}-\lambda Ax_\lambda^{(2)}\ni x_\lambda^{(1)},\ \cdots,\quad x_\lambda^{(n)}-\lambda Ax_\lambda^{(n)}\ni x_\lambda^{(n-1)},$$

has a solution $\{x_\lambda^{(1)}, x_\lambda^{(2)}, \cdots, x_\lambda^{(n)}\}$, where each $x_\lambda^{(\nu)}$ belongs to $B_{\rho(r,T)}\cap D(A)$, for every $x\in B_r\cap\overline{D(A)}$ and $\lambda\in(0, T/n]$.[2)] *Then*

$$\text{(2,1)}\quad \exp(tA)\cdot x=\lim_{n\to\infty}\{I-(t/n)A|_{B_{\rho(r,T)}}\}^{-n}x,\quad 0\leq t\leq T,\quad x\in B_r\cap\overline{D(A)},$$

exists in X for each $r, T>0$, and $\{\exp(tA); 0\leq t<+\infty\}$ is a semi-group

1) For the notion of "multiple-valued" operator, we refer to Kato [5], §2.

2) Since $0<\lambda<\omega(\rho(r, T))^{-1}$, one can write that $x_\lambda^{(1)}=(I-\lambda A|_{B_{\rho(r,T)}})^{-1}x$, $x_\lambda^{(2)}=(I-\lambda A|_{B_{\rho(r,T)}})^{-2}x, \cdots, x_\lambda^{(n)}=(I-\lambda A|_{B_{\rho(r,T)}})^{-n}x$.