215. *Integration of Equations of Imschenetsky Type by Integrable Systems*

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1. Introduction. Generalizing the method of integration due to Monge, the author gave a method of integration by integrable systems in [1] and [2]. Here we shall prove the following

Theorem. *Transform an equation of Imschenetsky type by one of the associated Imschenetsky transformations. Then the transformed equation is solved by integrable systems of order $n-1$, if and only if the original equation is solved by integrable systems of order n.*

This is a generalization of results in [1], [2] obtained for the Laplace transformation associated with a linear hyperbolic equation, and for the Imschenetsky transformation associated with an equation of Laplace type. In the second case the theorem was proved only for $n=1, 2$. In both the cases the author obtained the invariants of the equation whose vanishing is a necessary and sufficient condition in order that the equation may be solved by integrable systems of order n, and proved that the invariants for the original equation to be solved by integrable systems of order n are transformed to those for the transformed equation to be solved by integrable systems of order $n-1$. Here we shall prove the theorem directly, without obtaining the invariants of the equations.

2. Integrable systems of order *n*. Let us try to solve the Cauchy problem of an equation of type

$$s+f(x, y, z, p, q)=0, \tag{1}$$

integrating ordinary differential equations, in the space of $(x, y, z, p, q_1, \cdots, q_n)$. Here, $p=\partial z/\partial x$, $q=\partial z/\partial y$, $s=\partial^2 z/\partial x\partial y$, and $q_i=\partial^i z/\partial y^i$ $(q_1=q)$. The Cauchy problem in this space involving the derivatives of higher order is to find a two-dimensional submanifold which satisfies

$$\begin{aligned} dz-pdx-qdy&=dq_1+f_0dx-q_2dy=dq_2+f_1dx-q_3dy \\ &=\cdots=dq_{n-1}+f_{n-2}dx-q_ndy=0, \end{aligned} \tag{2}$$

and contains a given initial curve satisfying (2). Here, f_i is a function of $(x, y, z, p, q_1, \cdots, q_{i+1})$ defined inductively by

$$f_i=\left(G_i-f\frac{\partial}{\partial p}\right)f_{i-1} \quad (i\geqq 1), \quad f_0=f$$

with