213. On the Structure of Hyperfunctions with Compact Supports

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We discuss an analogue of the classical structure theorem of distributions on a compact set. We mainly treat the case of one variable ($n=1$). The case of several variables with some applications will be discussed by a somewhat different method in a paper under preparation (see [3]).

Theorem 1. *Let u be a hyperfunction of one variable with support in the interval $K=[a, b]$. Then u can be expressed as follows:*

$$u=J_1(D)\mu_1+J_2(D)\mu_2+J_3(D)\mu_3,$$

where μ_i, $i=1,2,3$ are measures with supports in $[a, b]$, and $J_i(D)$, $i=1,2,3$ are local operators with constant coefficients. (Local operators with constant coefficients are differential operators of infinite order in the theory of hyperfunctions; see, e.g., [1], § 2. On the operation of $J_i(D)$, the measures μ_i are considered as hyperfunctions.)

We prepare two lemmas. Let $\mathscr{B}[K]$ denote the space of hyperfunctions with support in K. Let $H_K(\zeta)$ denote the supporting function $\sup_{x\in K} \operatorname{Re}\langle x, i\zeta\rangle$ of K ($i=\sqrt{-1}$).

Lemma 2. *The Fourier transform $\tilde{u}(\zeta)$ of $u\in\mathscr{B}[K]$ is an entire function which satisfies the following growth condition:*

$$|\tilde{u}(\zeta)|\leqslant C\exp(|\zeta|/\varphi(|\zeta|)+H_K(\zeta)),$$

where $\varphi(r)$ is a monotonely increasing function of $r\geqslant 0$ and satisfies $\varphi(0)=1$, $\varphi(r)\to\infty$ when $r\to\infty$.

Proof. The following estimate for $\tilde{u}(\zeta)$ is well known:

$$|\tilde{u}(\zeta)|\leqslant C_\varepsilon\exp(\varepsilon|\zeta|+H_K(\zeta)) \qquad \text{for any } \varepsilon>0.$$

Put

$$\psi(r)=\sup_{|\zeta|=r}|\tilde{u}(\zeta)\exp(-H_K(\zeta))| \quad \text{and} \quad \psi_1(r)=r/\log(e+\psi(r)).$$

From the above estimates it is easily seen that $\psi_1(r)\to\infty$ when $r\to\infty$. Thus the function $\varphi(r)=\max\{\inf_{s\geqslant r}\psi_1(r), 1\}$ serves our purpose. q.e.d.

Lemma 3. *Assume that the function $\varphi(r)$ has the properties mentioned in Lemma 2. Then for any prescribed constants A, C, c_1, c_2 there exists a local operator $J(D)$ whose Fourier transform $J(\zeta)$ satisfies the following estimate from below:*

$$|J(\zeta)|\geqslant C\exp(A|\zeta|/\varphi(|\zeta|)) \qquad \textit{for } |\operatorname{Im}\zeta|\leqslant c_1+c_2|\operatorname{Re}\zeta|.$$

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