212. *Some Nonlinear Evolution Equations of Second Order*

By Masayoshi TSUTSUMI
Department of Applied Physics, Waseda University, Tokyo

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 13, 1971)

1. Introduction. Let H and W be two real separable Hilbert spaces and V be a real separable reflexive Banach space with $V\subset W\subset H$. Let V be dense in W and in H and the natural injections of V into W and of W into H be respectively continuous and compact. We identify H with its dual:

$$V\subset W\subset H\subset W^*\subset V^*$$

where W^* and V^* are the duals of W and V, respectively. The pairing between V and V^* is denoted by $(\ ,\ )$ and that of W and W^* by $\langle\ ,\ \rangle$.

We consider the following second order differential equation

$$u''+A(u)+Bu'=f \tag{1.1}$$

with initial conditions

$$u(0)=u_0, \qquad u'(0)=u_1, \tag{1.2}$$

where $u=u(t)$, $u'=du/dt$, $u''=d^2u/dt^2$ and data u_0, u_1, f are given.

Assume that the nonlinear operator $A: V\to V^*$ has the following properties:

1) A is hemicontinuous and $\|A(u)\|_{V^*}\leqq c\,\|u\|_V^{p-1}$, $p>1$, $c>0$.

2) A is monotone, i.e., $(A(u)-A(v),\ u-v)\geqq 0$, $\forall u, v\in V$.

3) $(A(u), u)=\|u\|_V^p$.

4) $A(u)$ is Fréchet differentiable at every $u\in V$.

5) $A(u)$ is strongly homogeneous of degree $p-1$ in the sense of Dubinskii [1], i.e., for every u, $\eta\in V$

$$(A'(u)\eta, u)=(A'(u)u, \eta)=(p-1)(A(u), \eta) \tag{1.3}$$

where $A'(u)$ is a Fréchet derivative.

Let $B: W\to W^*$ be a bounded linear operator associated with a bounded symmetric bilinear form $b(\cdot,\cdot)$ on W, i.e.,

$$|b(u, v)|\leqq\|u\|_W\,\|v\|_W, \qquad b(u, v)=b(v, u),$$
$$b(u, v)=\langle Bu, v\rangle, \qquad \forall u, v\in W,$$

such that

$$b(u, u)\geqq\alpha\,\|u\|_W^2-\beta\,\|u\|_H^2, \qquad \alpha, \beta>0, \tag{1.4}$$

and that if $u_n\to u$ weakly in W as $n\to\infty$,

$$\liminf_n b(u_n, u_n)\geqq b(u, u). \tag{1.5}$$

The main result of this note is the following theorem.

Theorem 1. *Suppose that* $u_0\in V$, $u_1\in H$ *and* $f\in L^2(0, T; H)$. *Then there exists at least one function* u *such that*