210. *Topological Completions and Realcompactifications*

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Throughout this paper by a space we shall mean a completely regular T_1-space. The completion of a given space X with respect to its finest uniformity is called the topological completion of X, according to Morita [5], denoted by μX. The following question has been raised by Comfort [1]: Is there a locally compact space with a realcompactification which is not even a k-space? An ingenious example has been suggested by the referee of the above paper, and it is described in [1]. The cardinality of the space in this example is $\aleph_2$. Negrepontis [7] constructed further a locally compact separable space of cardinality $\aleph_1$, assuming the continuum hypothesis, whose realcompactification is a Lindelöf non k-space. In § 1, concerning this question, we shall point out the fact that if X is a normal space satisfying the condition $(cc\to c)$ and if Y is a subspace of υX such that $X\subsetneqq Y$, then Y is not a k-space (Theorem 1.1 below) and moreover investigate the relation between μX and Hewitt realcompactification υX of a given space X concerning local compactness (Theorem 1.5 below). In § 2, firstly we shall prove that the relation $\mu(X\times Y)=\mu X\times \mu Y$ holds whenever $\upsilon(X\times Y)=\upsilon X\times \upsilon Y$ holds. When we consider, in general, those pairs of spaces X and Y for which $\upsilon(X\times Y)=\upsilon X\times \upsilon Y$ holds, we are involved in their cardinalities deeply, and Comfort [1] obtained interesting results about this relation under certain conditions for cardinality of space. But we shall show that analogous theorems to Comfort's main results hold without regard to the cardinality in connection with the topological completion (Theorem 2.3 below). In § 3, we consider the classes of spaces which are defined in terms of the relation $\mu(X\times Y)=\mu X\times \mu Y$ similarly to McAuthur [4].

§ 1. The local compactness and k-ness of μX and υX.

In this section the following theorems are useful for our discussion of the relation between μX and υX.

(M.1.1) (Theorem 2, Morita [5]). $X\subset \mu X\subset \upsilon X$.

(C.1.1) (Theorem 4.6, Comfort [1]). In order that υX be locally compact, it is necessary and sufficient that for each $p\in \upsilon X$ there exist pseudocompact subsets A and B of X for which $p\in \mathrm{Cl}_{\upsilon X} A$ and there exists $f\in C^*(X)$ such that $f=0$ on A and $f=1$ on $X-B$.

(C.1.2) (Theorem 4 (Hager-Johnson), Comfort [1]). Let U be an