209. *Hypersurfaces of a Euclidean Space* R^{4m}

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Introduction. K. Yano and M. Okumura [5] have shown that the existence of the so called (f, g, u, v, λ)-structure on hypersurfaces of an almost contact manifold and on submanifolds of codimension 2 of an almost Hermitian manifold.

D. E. Blair, G. D. Ludden and K. Yano [1] have studied complete hypersurfaces immersed in S^{2n+1} and showed that (1) if the Weingarten map of the immersion and f commute then the hypersurface is a sphere, and (2) if the Weingarten map K of the immersion and f satisfy $fK+Kf=0$ and the hypersurface is of constant scalar curvature, then it is a great sphere or $S^n \times S^n$.

On the other hand, Y. Y. Kuo [2] has shown the existence of an almost contact 3-structure on R^{4m+3} and that of a Sasakian 3-structure on S^{4m+3} and on the real projective space P^{4m+3}.

The main purpose of this paper is, after showing that an orientable hypersurface of a Hermitian manifold with quaternion structure admits an almost contact 3-structure (ϕ_i, ξ_i, η_i), $i=1, 2, 3$, to classify complete hypersurfaces of R^{4m} satisfying $\phi_i H - H\phi_i = 0$, $i=1, 2, 3$ and those satisfying $\phi_i H + H\phi_i = 0$, $i=1, 2, 3$. The results are:

Theorem 1. *Let N be a complete hypersurface of $R^{4m}(m \geqq 2)$. If the Weingarten map of the immersion and ϕ_i, $i=1, 2, 3$ commute, then N is one of the following*

(i) *a hyperplane,*

(ii) *a sphere,*

(iii) $R^{4t} \times S^{4s+3}$, $t+s=m-1$, $t \geqq 1$, $s \geqq 0$.

Theorem 2. *Let N be a complete hypersurface of $R^{4m}(m \geqq 1)$. If the Weingarten map H of the immersion and ϕ_i satisfy $\phi_i H + H\phi_i = 0$, then it is a hyperplane.*

For the case $m=1$ in Theorem 1, we have, as a corollary,

Corollary. *Let N be a complete hypersurface of R^4. If the Weingarten map of the immersion and ϕ_i, $i=1, 2, 3$ commute, then N is either a hyperplane or a sphere.*

1. Preliminaries. First, let $M=M^{4m}$ be a differentiable manifold with quaternion structure (Φ_1, Φ_2), where a quaternion structure is, by definition, a pair of two almost complex structures Φ_1, Φ_2 such that

(1) $$\Phi_1\Phi_2 + \Phi_2\Phi_1 = 0.$$