206. *Remark on Fixed Point of k-regular Mappings*

By Haruo MAKI
Department of Mathematics, Wakayama University, Wakayama

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 13, 1971)

The main purpose of this paper is to answer the question raised in [4]. The dilation D_k of Euclidean n-space R^n defined by $x \mapsto kx$ for some $k \in (0,1)$ can be extended uniquely to the n-sphere, $S^n = R^n \cup \{\infty\}$. If h is a homeomorphism of S^n of the same topological type as D_k, then h is regular except at two points. Kérekjartó [6], Homma and Kinoshita [2] showed the converse for $n=2$, $n=3$ respectively. Husch [3] extended Homma and Kinoshita's result for $n \geqq 6$. He [4] considered the topological characterization of the dilation in a separable infinite dimensional Fréchet space E (i.e. in a separable infinite dimensional locally convex complete linear metric space).

In [4], Husch has the following theorems. Let h be a homeomorphism of E (with metric d) onto itself.

Theorem (Husch [4]). *Suppose that h is k-regular at each point of E, $0<k<1$ (i.e. for each $\varepsilon>0$, there exists $\delta>0$ such that if $d(x,y)<\delta$, then $d(h^n(x), h^n(y))<k^n\varepsilon$ for each integer n).*

(1) ([4], Proposition 6, p. 4) *h has at most one fixed point.*

(2) ([4], Theorem 1, p. 2) *If the fixed point set of h, Fix (h), is not empty, then h has the topological type of a dilation D_k.*

(3) ([4], Theorem 2, p. 2) *If Fix (h) is empty, then h has the topological type of a translation.*

In this paper we prove the following:

Theorem 1. *If h is k-regular at each point of E, $0<k<1$, then h has a unique fixed point.*

Hence we can eliminate the hypothesis that *Fix*(h) be a non empty set in Husch's result (2).

Every separable infinite dimensional Fréchet space E is homeomorphic to the countable infinite product of lines [1]. Hence E is connected metric space. Thus we only show the following:

Lemma 2. *Let h be a k-regular mapping, ($0<k<1$), of a complete, connected metric space X onto itself. Then h has a unique fixed point.*

Before starting the proof, we recall the following definitions and some properties [5]. Let h be a continuous mapping in a metric space X. If for each $\varepsilon>0$, there exists $n \in I^+$ (positive integers) such that

$$d(h^m(x), h^m(y))<\varepsilon \qquad \text{for all } m \geqq n,$$