204. Remarks on the Asymptotic Behavior of the Solutions of Certain Third Order Non-Autonomous Differential Equations

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1. **Introduction.** The differential equations considered here are of the form

(1.1) $$\dddot{x} + \psi(t, x, \dot{x}, \ddot{x}) + \phi(t, x, \dot{x}) + c(t)f(x) = 0$$

(1.2) $$\dddot{x} + \psi(t, x, \dot{x}, \ddot{x}) + \phi(t, x, \dot{x}) + c(t)f(x) = e(t)$$

where ψ, ϕ, c, f and e are real valued functions. The dots indicate differentiation with respect to t and all solutions considered are assumed real.

In [3] Swick considered the behavior as $t \to \infty$ of solutions of the differential equations

(1.3) $$\dddot{x} + a\ddot{x} + g(x)\dot{x} + h(x) = e(t)$$

(1.4) $$\dddot{x} + p(t)\ddot{x} + q(t)g(\dot{x}) + h(x) = e(t)$$

where a is a positive constant. In [4] Swick also considered the asymptotic stability in the large, as $t \to \infty$, of the zero solution of the differential equations

(1.5) $$\dddot{x} + p(t)\ddot{x} + q(t)g(\dot{x}) + r(t)h(x) = 0$$

(1.6) $$\dddot{x} + f(x, \dot{x}, t)\ddot{x} + q(t)g(\dot{x}) + r(t)h(x) = 0.$$

Recently, in [1] T. Hara obtained some conditions under which all solutions of the equations

(1.7) $$\dddot{x} + a(t)\ddot{x} + b(t)\dot{x} + c(t)x = 0$$

(1.8) $$\dddot{x} + a(t)f(x, \dot{x})\ddot{x} + b(t)g(x, \dot{x})\dot{x} + c(t)x = 0$$

tend to the zero solution as $t \to \infty$.

In [5], the author established conditions under which all solutions of the non-autonomous equation (1.1) tend to the zero solution as $t \to \infty$.

In this note we investigate the asymptotic behavior of the solutions of the equation (1.2), and the asymptotic stability in the large of the zero solution of the equation (1.1) under a condition slightly weaker than that obtained in [5].

Many results have been obtained on the asymptotic property of autonomous equations of third order and many of these results are summarized in [2].

2. **Assumptions and Theorems.** We shall state the assumptions on the functions ψ, ϕ, f, c and e appeared in the equations (1.1) and (1.2).