128. On Some Separation Properties of a Function Algebra

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§1. In this paper we shall consider the relation among some separation property of a function algebra on a compact Hausdorff space and a property of a representing space of a strongly regular function algebra. According to the first one, we can eliminate the assumption "in a weak sense" from "approximately normal in a weak sense" and "approximately regular in a weak sense" of the theorem in the previous paper [5]. Recently D. Wilken [10] has shown that there exists no strongly regular function algebra on the closed unit interval I except $\mathcal{C}$ [I], according to the second one, we shall obtain a sufficient condition for a representing space $\mathcal{X}$ to satisfy that any strongly regular function algebra on $\mathcal{X}$ is nothing but $\mathcal{C}(\mathcal{X})$ itself.

Throughout this paper let $\mathcal{M}(\mathcal{A})$ be the maximal ideal space of a function algebra $\mathcal{A}$, $\Gamma(\mathcal{A})$ the Silov boundary and Cho $(\mathcal{A})$ the Choquet boundary of $\mathcal{A}$, respectively. Further, let, for any subset S of $\mathcal{X}$ (or $\mathcal{M}(\mathcal{A})$), $f(S)$ be the set $\{f(x)\,;\, x \in S\}$, $\mathcal{A}|S$ the restriction of $\mathcal{A}$ to S and A_s the uniform closure of $\mathcal{A}|S$ in $\mathcal{C}(S)$.

§2. We owe the following definition to D. Wilken [9].

Definition. A function algebra $\mathcal{A}$ is said to be approximately regular on $\mathcal{X}$, iff, for each point p in X and each closed set K not containing p and for any positive number ε, there is a function f in $\mathcal{A}$ such that $f(p)=1$ and $|f(y)<\varepsilon$ for y in K. $\mathcal{A}$ is said to be approximately normal on $\mathcal{X}$ iff, for any two disjoint closed subsets K_1 and K_2 and for any $\varepsilon>0$, there is a function f in $\mathcal{A}$ such that $|f(x)-1|<\varepsilon$ on K_1 and $|f(y)|<\varepsilon$ on K_2.

Let us define a new separation property of a function algebra as follows.

Definition. A function algebra $\mathcal{A}$ satisfies the condition $(*)$ on a closed subset S of $\mathcal{M}(\mathcal{A})$ iff for any connected closed subset K in S, the Silov boundary of $\mathcal{A}_K$ is K.

It is evident that if $\mathcal{A}$ is approximately normal, then $\mathcal{A}$ is approximately regular and if $\mathcal{A}$ is approximately regular, then $\mathcal{A}$ satisfies the condition $(*)$. We know by the following example that, in general, $(*)$ is weaker than approximate regularity [5].

Example. Let $\mathcal{X}=\{z\,;\,|z|\leqq 1\}$, $T=\{z\,;\,|z|=1\}$, $\mathcal{A}=\{f\in\mathcal{C}(\mathcal{X})\,;$ for