122. *Remarks on the Asymptotic Behavior of the Solutions of Certain Non-Autonomous Differential Equations*

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(Comm. by Kenjiro SHODA, M. J. A., Sept. 12, 1972)

1. Introduction. In this paper we consider the asymptotic behavior of the solutions of the non-autonomous, nonlinear differential equation;

(1.1) $$\dot{x}=A(t)x+f(t,x)$$

where x, f are n-dimensional vectors, $A(t)$ is a bounded continuously differentiable $n\times n$ matrix for $t\geqq 0$, and $f(t,x)$ is a continuous in (t,x) for $t\geqq 0$, $\|x\|<\infty$, here $\|\cdot\|$ denotes an Euclidean norm. And consider

(1.2) $$\dddot{x}+a(t)\ddot{x}+b(t)g(x,\dot{x})\dot{x}+c(t)h(x)=p(t,x,\dot{x},\ddot{x})$$

where $a(t), b(t), c(t)$ are positive, continuously differentiable and g, h, p are continuous real-valued functions depending only on the arguments shown, the dots indicate the differentiation with respect to t. In this note, certain conditions are obtained under which all solutions of (1.1) tend to zero as $t\to\infty$.

In [6], the author studied the asymptotic behavior of the solution of the equation

(1.3) $$\dddot{x}+a(t)f(x,\dot{x})\ddot{x}+b(t)g(x,\dot{x})\dot{x}+c(t)h(x)=e(t)$$

under the assumptions that $|a'(t),|b'(t)|,|c'(t)|$ and $e(t)$ are integrable and suitable conditions on f, g, h. Here we assume the condition that

$$\limsup_{t\to\infty}\frac{1}{t}\int_0^t\{|a'(s)|+|b'(s)|+|c'(s)|\}ds$$

has an infinitesimal upper bound,

to prove the every solution of (1.2) tends to zero as $t\to\infty$. Conditions on $p(t,x,y,z)$ are also relaxed. Theorem 2 generalizes the Ezeilo's result [5] in which he considered the equation

(1.4) $$\dddot{x}+a_1\ddot{x}+a_2\dot{x}+f_3(x)=p_1(t,x,\dot{x},\ddot{x}),$$

where a_1, a_2 are positive constants.

The main tool used in this work is Lemma 1 which is a specialization of the result obtained by F. Brauer [1]. Using this Lemma and Liapunov functions, we shall obtain Theorem 1 and Theorem 2. Lemma 1 is especially convenient to study the non-autonomous differential equations.

The author wishes to express his hearty thanks to Dr. M. Yamamoto of Osaka University for his invaluable advices and encouragements.