156. *On Extension of Continuous Mappings on Countably Paracompact Normal Spaces*

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The generalisations of extension theorem on continuous functions were developed by R. Arens (1), C. H. Dowker (5), J. Dugundji (6), and C. Kuratowski (12). A space is called countably paracompact if every countable open covering has a locally finite refinement. Hence countably paracompact spaces are a generalisation of paracompact spaces. C. H. Dowker and M. Katětov (11) have shown that countably paracompact normal spaces have many important properties. Recently B. J. Ball (2) proved that every linearly ordered space is countably paracompact.

In this note, we shall prove an extension theorem of continuous mappings on countably paracompact normal space, and a theorem of ANR as its application.

§1. An extension theorem on countably paracompact normal spaces

Theorem 1. Let A be a closed subset of countably paracompact normal space X, and f be a continuous map on A with values in a separable Banach space S. Then f may be extended to a map (continuous) $\bar{f}$ of X into S.

The proof will be technically the same for the collectionwise normal space (C. H. Dowker (5)). We shall first some lemmas which will be used in the proof of Theorem 1.

Lemma 1. Any Banach space is countably paracompact normal.

Proof. Since a metric space is paracompact normal (see A. H. Stone (15)), any Banach space is countably paracompact normal.

Lemma 2. If α is a locally finite covering of a normal space X, there is a canonical map ϕ of X into the nerve $|X_\alpha|$ of X.

For the detail, see C. H. Dowker (4), p. 202 or S. Eilenberg and N. Steenrod (7), p. 286.

Proof (the idea only). Let β be a shrinkable covering of α. Since X is normal, such a covering β exists. (See S. Lefschetz (14), p. 26.) Therefore by the decomposition of unity, there are continuous maps $\phi_\nu(x)$ such that

1) $\phi_\nu(x) \geqslant 0$,

2) $\sum\phi_\nu(x)=1$ for $x \in X$,