151. *On Spaces Having the Weak Topology with Respect to Closed Coverings. II*

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In the first paper under this title [4] we have introduced the following notion. Let X be a topological space and $\{A_\alpha\}$ a closed covering of X. Then X is said to *have the weak topology with respect to* $\{A_\alpha\}$, if the union of any subcollection $\{A_\beta\}$ of $\{A_\alpha\}$ is closed in X and any subset of $\bigcup_\beta A_\beta$ whose intersection with each A_β is open relative to the subspace topology of A_β is necessarily open in the subspace $\bigcup_\beta A_\beta$.

Any CW-complex (cf. [5]) has the weak topology with respect to the closed covering which consists of the closures[1] of all the cells. As another example we remark that a topological space has always the weak topology with respect to any locally finite closed covering.[2]

The purpose of this paper is to establish the following theorem.

Theorem 1. *Let X be a topological space having the weak topology with respect to a closed covering $\{A_\alpha\}$. Then X is paracompact and normal if and only if each subspace A_α is paracompact and normal.*

Thus if X has the weak topology with respect to a closed covering $\{A_\alpha\}$, each of the following properties for all subspaces A_α implies the same property for X: (1) normality, (2) complete normality, (3) perfect normality, (4) collectionwise normality, (5) paracompactness and normality, (6) countable paracompactness and normality. On the other hand, local compactness or metrizability[3] for all A_α does not necessarily imply the same property for X.

§1. Lemmas

Lemma 1. *Let A be a closed subset of a paracompact and normal space X. If $\{G_\alpha\}$ is a locally finite system in A which consists of open F_σ-sets G_α of A, then there exists a locally finite system $\{H_\alpha\}$ of open F_σ-sets of X with the following properties:*

1) The closure of a cell e should be understood here as that in the complex, that is, as the intersection of all subcomplexes containing e.

2) From Theorem 1 below it follows immediately that a topological space which is the union of a locally finite collection of closed, paracompact, normal subspaces is paracompact and normal; this proposition is remarked also by E. Michael [2].

3) We have learned that the latter proposition given in the remark at the end of [4] was already proved by J. Nagata in his paper: On a necessary and sufficient condition of metrizability, Jour. Inst. Polytech. Osaka City Univ., Ser. A, **1**, 93–100 (1950).