Now since C is convex, $\lambda x_n+(1-\lambda)x_m$ is in C, so that

$$\|x_n-x_m\|^2<\frac{(\rho+\varepsilon)^2}{\lambda(1-\lambda)}-\frac{\rho^2}{\lambda(1-\lambda)}<\frac{(2\rho+\varepsilon)\varepsilon}{\alpha^2}.$$

Let $x_0=\lim_{n\to\infty}x_n$, then x_0 is in C since C is closed, and it follows from the continuity of the norm that $\|x_0\|=\rho$. It is an immediate consequence of Lemma 1 and the condition $(*)$ that the element x_0 is unique.

We shall now proceed to prove the above-mentioned statement. Let x_0 be an element of E which does not belong to M; then the set $\{y-x_0 \mid y\in M\}$ is clearly convex and closed, so by Lemma 2 there is a unique element y_0 such that $\|y_0-x_0\|\leqq\|y-x_0\|$ for all $y\in M$.

It is easy to see that for all $y\in M$, we have

$$\|y-y_0\|\leqq\|y-x_0\|.$$

In fact, if $\|y-y_0\|$ is greater than $\|y-x_0\|$ for some $y\in M$, then in virtue of Lemma 1 there exists a λ, $0<\lambda<1-\alpha$, such that

$$\|\lambda y+(1-\lambda)y_0-x_0\|<\|y_0-x_0\|$$

which is a contradiction since $\lambda y+(1-\lambda)y_0$ is in M.

Now we define $\quad I^*(x)=I(y)+\lambda y_0=y+\lambda y_0$

for any $\quad x=y+\lambda x_0,\ y\in M,\ \lambda\in R$.

Then it is clear that I^* is linear and an extension of I to $M+Rx_0$, and hence it remains only to prove the continuity of I^* and that the norm is 1. For that matter the relation

$$\|y+\lambda y_0\|=|\lambda|\cdot\|\lambda^{-1}y+y_0\|$$

holds for $\lambda\neq 0$.

On the other hand, $\quad \|\lambda^{-1}y+y_0\|\leqq\|-\lambda^{-1}y-x_0\|$,

and so $\quad \|y+\lambda y_0\|\leqq\|y+\lambda x_0\|$,

which guarantees the continuity of I^* and shows the norm is 1. Thus we have reached the desired conclusion.

Additions and Corrections to Shouro Kasahara: "A Note on f-completeness"

(Proc. Japan Acad., **30**, No. 7, 572–575 (1954))

Pages 572–573, delete "Proposition 2".

Page 574, delete "Proposition 6".

Page 574, line 19 from foot, for "mapping of W, we have $p(I^*(x))\leqq p(x)$ for any $p\in(p_\alpha)$ and $x\in E$." read "mapping of W, concerning to $p\in(p_\alpha)$, we have $p^*(I(x))\leqq p(x)$ for any $x\in E$.".

Page 574, lines 26–29, delete "Now, since$\cdots$inequality $(*)$ for u^*.".

Pape 574, line 10 from foot, for "for any $p\in(p_\alpha)$ there is" read "there exist a $p\in(p_\alpha)$ and".

Page 574, line 2 from foot, for "*same a*" read "*same p and a*".