16. On Newman Algebra. III

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Introduction

There are many kinds of postulates for Boolean algebra and Boolean ring as shown by D. G. Miller[1] in his recent paper. The same holds also for Newman algebra which we considered in previous papers I, II.[2] (We remark that we call Newman algebra the algebraic system which is the direct union of a Boolean algebra (=Boolean lattice) and a Boolean ring (with unity), the latter satisfying also the associative law for multiplication, whereas usually the validity of this last law is not assumed in the definition of Newman algebra.) We have given two postulate-sets I*, II* for our Newman algebra respectively in I, II; now we propose to give another one III*.

In §1 below we shall give the complete list of Set III*, and prove the equivalence of Set III* with Set I*, wherewith the equivalence of all Sets I*, II*, III* will be established as we have already proved II* as equivalent with I*. In §2 we give the independence proofs. The eight-element system we use has been constructed by the same method as we have explained in another paper.[3]

We shall give here the difference of our Set III* from Set I* and Set II*. We introduce a new postulate

$$6'.\quad a(b+c)=ca+ba$$

which will replace 3. $a+b=b+a$ and 6. $a(b+c)=ab+ac$ of Set I*, and 6. $a(b+c)=ab+ac$ and 8′. $a+b'b=b'b+a$ of Set II*. The form of postulate 6′ was suggested by the postulate $a[(b+c)+d]=a(d+c)+ab$ of Miller.

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1. Postulates of Set III*

Our postulates are the propositions below on a *class K*, two *binary operation* $+$, $\times$ and a *unary operation* $'$ (in the postulates that are not existence postulates supply the condition: *if the elements indicated are in K*). It is to be remarked that the unary operation $'$ is not required to be single-valued in our postulates.