44. Probabilities on Inheritance in Consanguineous Families. XIII

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X. Combinations through extreme consanguineous marriages, 2

1. Definitions of quantities

In the present chapter we shall supplement the results on special combinations previously postponed as an intermediate extreme case. We first attempt to determine the probability of mother-descendants combination of the form

$$(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)_{(1\nu;\,0)_t|1\nu}\equiv(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)_{1\nu_1;\,0|\dots|1\nu_t;\,0|1\nu_{t+1}},$$

the generation-numbers $\nu_r\,(1\leqq r\leqq t+1)$ being supposed greater than unity.

We shall collect previously in the present section the definition of several quantities which will become necessary for establishing general formulas.

We first define a quantity $\mathfrak{R}^{\backprime}$ by

$$\mathfrak{R}^{\backprime}(\xi_1\eta_1,\ \xi_2\eta_2)=2\mathfrak{R}^*(\xi_1\eta_1,\ \xi_2\eta_2)-\bar{A}_{\xi_1\eta_1}Q(\xi_1\eta_1;\ \xi_2\eta_2).$$

We next introduce two quantities $\mathfrak{S}^{\backprime}$ and $\mathfrak{S}^{\backprime\backprime}$ by means of

$$\begin{aligned}16\sum\mathfrak{S}^*(\alpha\beta;\ ab,\ cd)\varepsilon(ab,\ cd;\ \xi_1\eta_1)E(ab,\ cd;\ \xi_2\eta_2)\\ =14\mathfrak{S}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-3S(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-\mathfrak{S}^{\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\\ =12\mathfrak{S}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-2S(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)+\mathfrak{S}^{\backprime\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\end{aligned}$$

and further two quantities $\mathfrak{T}^{\backprime}$ and $\mathfrak{T}^{\backprime\backprime}$ by means of

$$\begin{aligned}16\sum\mathfrak{T}^*(\alpha\beta;\ ab,\ cd)\varepsilon(ab,\ cd;\ \xi_1\eta_1)E(ab,\ cd;\ \xi_2\eta_2)\\ =14\mathfrak{T}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-3T(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-\mathfrak{T}^{\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\\ =12\mathfrak{T}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-2T(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)+\mathfrak{T}^{\backprime\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).\end{aligned}$$

There then hold evidently the relations

$$\begin{aligned}\mathfrak{S}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\tfrac{1}{2}\{S(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)+\mathfrak{S}^{\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\\ +\mathfrak{S}^{\backprime\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\},\\ \mathfrak{T}^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\tfrac{1}{2}\{T(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)+\mathfrak{T}^{\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\\ +\mathfrak{T}^{\backprime\backprime}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\}.\end{aligned}$$

We finally introduce two quantities V^* and W^* by

$$V^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=V(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-\bar{A}_{\xi_1\eta_1}Q(\alpha\beta;\ \xi_2\eta_2)-S(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2),$$

$$W^*(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=W(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-2\bar{A}_{\xi_1\eta_1}Q(\alpha\beta;\ \xi_2\eta_2)-T(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

However, though we have not yet explicitly noticed, there holds a remarkable identity

$$2V(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-W(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=2S(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)-T(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$