43. *Probabilities on Inheritance in Consanguineous Families. XII*

By Yûsaku KOMATU and Han NISHIMIYA
Department of Mathematics, Tokyo Institute of Technology
(Comm. by T. FURUHATA, M.J.A., March 12, 1955)

IX. Combinations through extreme consanguineous marriages, 1

1. Mother-descendants combinations

The main purpose of the present and the subsequent chapters is to deal with a mother-descendants combination through repeated extreme consanguineous marriages, of which the reduced probability is designated by

$$\kappa_{(\mu\nu;0)_t|\mu\nu}(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)\equiv\kappa_{\mu_1\nu_1;0|\cdots|\mu_t\nu_t;0|\mu_{t+1}\nu_{t+1}}(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2),$$

and also several related combinations.[1] Among such a combination the most extreme case $\mu_r=\nu_r=1$ for $1\leqq r\leqq t$ has been observed already in the preceding chapter. For the remaining cases we shall restrict ourselves to consider both canonical cases $\mu_r=1<\nu_r$ and $\mu_r,\nu_r>1$ for $1\leqq r\leqq t$, so that some intermediate cases will be omitted.

We first consider the probability $\kappa_{(\mu\nu;0)_t|\mu\nu}$ for the generic case, namely with $\mu_r,\nu_r>1$ for $1\leqq r\leqq t$. The reduced probability is evidently given by a recurrence equation

$$\kappa_{(\mu\nu;0)_t|\mu\nu}(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)=\sum\kappa_{(\mu\nu;0)_{t-1}|\mu_t\nu_t}(\alpha\beta;ab,cd)\varepsilon_\mu(ab,cd;\xi_1\eta_1)\varepsilon_\nu(ab,cd;\xi_2\eta_2)$$

where the summation extends over all the possible pairs of genotypes A_{ab} and A_{cd}.

Under the initial condition expressed by

$$\kappa_{\mu\nu}(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)=\bar{A}_{\xi_1\eta_1}\bar{A}_{\xi_2\eta_2}+2^{-\mu+1}\bar{A}_{\xi_2\eta_2}Q(\alpha\beta;\xi_1\eta_1)+2^{-\nu+1}\bar{A}_{\xi_1\eta_1}Q(\alpha\beta;\xi_2\eta_2)$$
$$+2^{-\lambda}T(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)\qquad(\lambda=\mu+\nu-1),\text{[2]}$$

it can be shown that *the recurrence equation is solved by*

$$\kappa_{(\mu\nu;0)_t|\mu\nu}(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)=\bar{A}_{\xi_1\eta_1}\bar{A}_{\xi_2\eta_2}$$
$$+\varLambda_t\{2^{-\mu+1}\bar{A}_{\xi_2\eta_2}Q(\alpha\beta;\xi_1\eta_1)+2^{-\nu+1}\bar{A}_{\xi_1\eta_1}Q(\alpha\beta;\xi_2\eta_2)\}$$
$$+x_{t+1}\bar{A}_{\xi_1\eta_1}Q(\xi_1\eta_1;\xi_2\eta_2)+y_{t+1}S(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2)+z_{t+1}T(\alpha\beta;\xi_1\eta_1,\xi_2\eta_2),$$

where the coefficients are defined by

$$\varLambda_t=\prod_{r=1}^{t}(2^{-\mu_r}+2^{-\nu_r}),\qquad\lambda_r=\mu_r+\nu_r-1,$$

---

1) Previous papers under the same title have been published in Proc. Japan Acad. **30** (1954), 42-52, 148-155, 236-247, 636-654. For full details, cf. Y. Komatu and H. Nishimiya, Probabilistic investigations on inheritance in consanguineous families. Bull. Tokyo Inst. Tech. (1954), 1-66, 67-152, 153-222 et seq.

2) With respect to several quantities involved in the formula, cf. the papers cited in 1).