28. On the Riesz Logarithmic Summability of the Conjugate Derived Fourier Series. I

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1. Let $f(x)$ be an integrable function with period 2π and its Fourier series be

(1.1) $$f(x) \sim a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \equiv \sum_{n=0}^{\infty} A_n(x).$$

We call the series

(1.2) $$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) \equiv \sum_{n=1}^{\infty} B_n(t),$$

$$\sum_{n=1}^{\infty} n(b_n \cos nx - a_n \sin nx) = \sum_{n=1}^{\infty} A_n'(t)$$

and

(1.3) $$\sum_{n=1}^{\infty} n(a_n \cos nx + b_n \sin nx) = \sum_{n=1}^{\infty} nA_n(x)$$

conjugate series, derived series and conjugate derived series of (1.1), respectively.

The infinite series $\sum a_n$ is said to be summable by Riesz's logarithmic mean of order α, or simply summable (R, log, α), to sum s, provided that

$$R_\alpha(\omega) = \frac{1}{(\log \omega)^\alpha} \sum_{n<\omega} (\log \omega/n)^\alpha a_n$$

tends to a limit s, as $\omega \to \infty$.

The summability by Riesz's logarithmic means of the Fourier series was treated by Hardy [1], Takahashi [3], and Wang [4], [5], [6]. Wang has proved the Riesz summability analogue of Bosanquet's theorem concerning Cesàro summability of Fourier series. This theorem was extended to the derived Fourier series by Matsuyama [2]. In this paper we shall prove the analogue for the conjugate derived Fourier series and some related theorems.

We shall introduce some notations. Let us put

$$g_0(t) = g(t),$$

$$g_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_t^\pi \left(\log \frac{u}{t}\right)^{\alpha-1} \frac{g(u)}{u} du \qquad (\alpha > 0).$$

Then $g_\alpha(t) \Big/ \left(\log \frac{1}{t}\right)^\alpha$ is called the Riesz logarithmic mean of $g(t)$ of order α. If the Riesz logarithmic mean of $g(t) - s$ tends to zero as $t \to 0$, then we write

$$\lim_{t \to 0} g(t) = s \ (R, log, \alpha).$$