24. On the Strong Summability of the Derived Fourier Series. II

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1. B. N. Prasad and U. N. Singh [1] have proved the following

Theorem 1. *Let $f(t)$ be a continuous function of bounded variation, with period 2π, and let*

$$g_x(u)=g(u)=f(x+u)-f(x-u)-2us,$$

then, if

$$(1)\qquad \int_0^t |dg(u)|=O\left[t\Big/\left(\log\frac{1}{t}\right)^{1+\varepsilon}\right]\qquad (t\to 0)$$

for a positive ε, then the derived Fourier series of $f(t)$ is strongly summable (or H_1-summable) to s at x, that is

$$(2)\qquad \lim_{n\to\infty}\frac{1}{n}\sum_{\nu=1}^{n}|\tau_\nu(x)-s|=0$$

$\tau_n(x)$ being the n-th partial sum of the derived Fourier series of $f(x)$.

In the first paper [2], one of us proved that under the assumption of Theorem 1[1)]

$$(3)\qquad \lim_{n\to\infty}\frac{1}{n}\sum_{\nu=1}^{n}|\tau_\nu(x)-s|^k=0,$$

for any $k>0$. But in its proof it is used, without stating explicitly, that the summability (H_k) is the local property for the derived Fourier series of $f(x)$. This is true by Wiener's theorem (A. Zygmund [6], p. 221).

We shall now consider an extension of Theorem 1 in the case $k\leqq 2$. In fact we shall prove

Theorem 2. *If*

$$(4)\qquad \int_0^t |dg(u)|=O\left[t\Big/\left(\log\frac{1}{t}\right)^{\alpha}\right]\qquad (t\to 0),$$

then

$$\lim\frac{1}{n}\sum_{\nu=1}^{n}|\tau_\nu(x)-s|^2=0\qquad \textit{for}\quad \alpha>1/2.$$

This is the analogue of Wang's theorem for Fourier series [3]. We can also prove the following

Theorem 3. *In Theorem 2, if the condition (4) is replaced by*

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1) In [2], $\tau_\nu^*(x)$ may be replaced by $\tau_\nu(x)$ and the last section, containing Theorems 3 and 4, must be omitted.