77. *Integrations on the Circle of Convergence and the Divergence of Interpolations. I*

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Let the points

$$\text{(P)} \qquad \begin{cases} z_1^{(1)} \\ z_1^{(2)},\ z_2^{(2)} \\ z_1^{(3)},\ z_2^{(3)},\ z_3^{(3)} \\ \cdots\cdots \\ z_1^{(n)},\ z_2^{(n)},\ z_3^{(n)},\ \dots,\ z_n^{(n)} \\ \cdots\cdots \end{cases}$$

which do not lie exterior to the unit circle $C: |z|=1$, satisfy the condition that the sequence of

$$\frac{w_n(z)}{z^n}=(z-z_1^{(n)})(z-z_2^{(n)})\cdots(z-z_n^{(n)})/z^n$$

converges to a function $\lambda(z)$ single valued, analytic, and non-vanishing for z exterior to C, and uniformly for any finite closed set exterior to C, that is

$$\text{(C)} \qquad \lim_{n\to\infty}\frac{w_n(z)}{z^n}=\lambda(z)\neq 0 \quad \text{for} \quad |z|>1.$$

Let $f(z)$ be a function single valued and analytic within the circle $C_\rho: |z|=\rho>1$ but not analytic on C_ρ. Then the sequence of polynomials $P_n(z;f)$ of respective degrees n which interpolate to $f(z)$ in all the zeros of $w_{n+1}(z)$ is known to be

$$\text{(I)} \qquad P_n(z;f)=\frac{1}{2\pi i}\int_{C_R}\frac{w_{n+1}(t)-w_{n+1}(z)}{w_{n+1}(t)}\,\frac{f(t)}{t-z}\,dt, \quad (1<R<\rho).$$

It is known that the sequence of polynomials $P_n(z;f)$ converges to $f(z)$ throughout the interior of the circle C_ρ, and uniformly for any closed set interior to C_ρ. But the divergence of $P_n(z;f)$ at every point exterior to C_ρ is not established in general.

This problem is seen in the paper by Walsh: *The divergence of sequences of polynomials interpolating in roots of unity*; Bulletin of the American Mathematical Society, 1936, Vol. 42, p. 715. And that is treated in the following papers by the author.

T. Kakehashi: *On the convergence-region of interpolation polynomials*; Journal of the Mathematical Society of Japan, 1955, Vol. 7, p. 32.

T. Kakehashi: *The divergence of interpolations*. I, II, III; Proceedings of the Japan Academy, 1954, Vol. 30, Nos. 8, 9, and 10.

In this paper, we consider a certain type of integrations on the convergence-circle of a function, which belongs to a certain class of