123. *The Decomposition of Coefficients of Power-series and the Divergence of Interpolation Polynomials*

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Let $f(z)=\sum_{n=0}^{\infty}a_nz^n$ be a function single valued and analytic within the circle $C_\rho:|z|=\rho>0$ but not analytic regular on C_ρ, and $S_n(z)$ be partial sums of the power series of respective degrees n, that is $S_n(z)=\sum_{k=0}^{n}a_kz^k$. Then it is known that the sequence of polynomials $S_n(z)$ of respective degrees n converges to $f(z)$ throughout the interior of the circle C_ρ, uniformly for any closed set interior to C_ρ, and diverges at every point exterior to C_ρ as n tends to infinity. And moreover, we have

$$\overline{\lim}_{n\to\infty}|S_n(z)|^{\frac{1}{n}}=\frac{|z|}{\rho}\qquad\text{for } z \text{ exterior to } C_\rho.$$

Above properties can be generalized to the sequence of polynomials found by interpolation to $f(z)$ in the points which satisfy a certain condition. (T. Kakehashi: *On the convergence-region of interpolation polynomials*, Journal of the Mathematical Society of Japan, 1955, Vol. 7).

In this paper, we consider the divergence property of the sequence found by interpolation in the set of points more generalized than that considered in the above paper.

Let the sequence of points

$$\text{(P)}\qquad\begin{cases} z_1^{(1)} \\ z_1^{(2)},\ z_2^{(2)} \\ z_1^{(3)},\ z_2^{(3)},\ z_3^{(3)} \\ \cdots\cdots \\ z_1^{(n)},\ z_2^{(n)},\ z_3^{(n)},\ldots,\ z_n^{(n)} \\ \cdots\cdots \end{cases}$$

which do not lie exterior to the unit circle $C:|z|=1$, satisfy the condition that the sequence of

$$\frac{w_n(z)}{z^n}=\frac{(z-z_1^{(n)})(z-z_2^{(n)})\cdots(z-z_n^{(n)})}{z^n}$$

converges to a function $\lambda(z)$, single valued, analytic, and non-vanishing for z exterior to C, and uniformly for any finite closed points set exterior to C, that is

$$\text{(C)}\qquad \lim_{n\to\infty}\frac{w_n(z)}{z^n}=\lambda(z)\neq 0\qquad\text{for } |z|>1.$$

Let $f(z)$ be a function single valued and analytic throughout the interior of the circle $C_\rho:|z|=\rho>1$ but not analytic regular on