# 122. On the Convergence Character of Fourier Series

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(Comm. by Z. SUETUNA, M.J.A., Oct. 12, 1955)

1. Let $f(x)$ be an integrable function with period $2\pi$ and $s_n(x)$ be the $n$th partial sum of Fourier series of $f(x)$.

Recently, S. Izumi[1] has proved the following theorem:

*If $f(x)$ belongs to the Lip $\alpha(0<\alpha\leqq 1)$ class, then the series*[2]

$$\sum_{n=1}^{\infty} |s_n(x)-f(x)|^2/n^{\beta}(\log n)^{\gamma}$$

*converges uniformly, where $\beta=1-2\alpha$ and $\gamma>1$ or $>2$ according as $0<\alpha<1/2$ or $1/2\leqq\alpha<1$.*

The object of this paper is to prove the following theorem, which may be partially more general than the above theorem:

**Theorem 1.** *If $f(x)$ belongs to the Lip $\alpha(0<\alpha<1/2)$ class then the series*

$$\sum_{n=1}^{\infty} \frac{|s_n(x)-f(x)|^k}{n^{\delta}(\log n)^{\gamma}}$$

*converges uniformly, where $\delta=1-k\alpha$, $\gamma>1$, $1>k\alpha$, and $k>0$.*

**Theorem 2.**[3] *If $f(x)$ belongs to the Lip $\alpha$ class and if $k\alpha=1$, then the series*

$$\sum_{n=1}^{\infty} \frac{|s_n(x)-f(x)|^k}{(\log n)^{\tau}}$$

*converges uniformly, where $\tau>(1-\alpha)/\alpha$ and $k\geqq 2$.*

2. For the proof of the theorem we need the following lemma:

**Lemma 1.** *Under the condition of Theorem 1, we have*

$$\sum_{\nu=1}^{n} |s_\nu(x)-f(x)|^k=O(n^{1-k\alpha}), \tag{2.1}$$

*uniformly.*

**Lemma 2.** *Under the condition of Theorem 2, we have*

$$\sum_{\nu=1}^{n} |s_\nu(x)-f(x)|^k=O([\log n]^{k-1}),$$

*uniformly.*

**Proof of Lemma 1.**[4] We have

$$I=\Big(\sum_{\nu=1}^{n} |s_\nu(x)-f(x)|^k\Big)^{1/k}$$

1) S. Izumi: Proc. Japan Acad., **31**, 257–260 (1955).

2) We suppose $1/(\log n)=1$ for $n=1$.

3) This theorem was suggested by Mr. I. Oyama.

4) Cf. A. Zygmund: Trigonometrical series, p. 238, and T. Tsuchikura: Mathematica Japonicae, **1**, 1–5 (1949).