120. Lacunary Fourier Series. II

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1. M. E. Noble [1] has proved the following

Theorem N. *If the Fourier series of* $f(t)$ *has a gap* $0<|n-n_k|\leqq N_k$ *such that*

$$\lim N_k/\log n_k=\infty$$

and $f(t)$ *satisfies a Lipschitz condition of order* α, *where* $\frac{1}{2}<\alpha<1$, *in some interval* $|x-x_0|\leqq\delta$. *Then*

$$\sum(|a_{n_k}|+|b_{n_k}|)<\infty,$$

where a_{n_k}, b_{n_k} *are non-vanishing Fourier coefficients of* $f(t)$.

As a continuation of the first paper [2] we treat absolute convergence of the Fourier series with a certain gap and satisfying some continuity condition at a point (Theorems 3 and 4).

We need following theorems and lemmas in [2].

Lemma 1. *Let* (δ_m) *be a sequence tending to zero and let* $n=[4em/\delta_m]$. *Then there exists a trigonometrical polynomial* $T_n(x)$ *of degree not exceeding* n *with constant term 1 such that*[1]

(i)	$\lvert T_n(x)\rvert\leqq A/\delta_m$,	*for all* x,
(ii)	$\lvert T_n(x)\rvert\leqq An/\delta_m e^m$,	*for* $\delta_m\leqq\lvert x\rvert\leqq\pi$,
(iii)	$\lvert T_n'(x)\rvert\leqq An/\delta_m$,	*for all* x,
(iv)	$\lvert T_n'(x)\rvert\leqq A(n^2/\delta_m e^m+1/x^2)$,	*for* $\lambda\delta_m\leqq\lvert x\rvert\leqq\pi$, $\lambda>1$,[2]
(v)	$\lvert T_n''(x)\rvert\leqq An^2/\delta_m$,	*for all* x,

where A *denotes an absolute constant.*

Theorem 1. *Let* $0<\alpha<1$ *and* $0<\beta<min(1-\alpha, (2-\alpha)/3)$. *If*

$$k^{2/(2-\alpha-3\beta)}<n_k<e^{2k/(2+\alpha+\beta)},$$

$$|n_{k\pm1}-n_k|>4ekn_k^{\beta}$$

and

$$\frac{1}{h^{\beta}}\int_0^{\beta}|f(t)-f(t\pm h)|\,dt=O(h^{\alpha}), \tag{1}$$

$$\frac{1}{\tau}\int_0^{\tau}|f(t)-f(t\pm h)|\,dt=O(1), \qquad \textit{unif. in } \tau\geqq h^{\beta}, \tag{2}$$

then

$$a_{n_k}=O(n_k^{-\alpha}), \quad b_{n_k}=O(n_k^{-\alpha}). \tag{3}$$

Lamma 2. *Let* (δ_m) *be a sequence tending to zero and let* $n=[4me^{1-m\delta'm/\delta_m}/\delta_m]$. *Then there exists a trigonometrical polynomial*

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1) A denotes an absolute constant which is not the same in different occurrences.

2) λ may be taken as near 1 as we like when m is sufficiently large.