119. Generalization of the Concept of Cohomology of Finite Groups

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The aim of the present note is to sketch foundations to establish relations between the cohomology theory and the theory of representations of finite groups. It is obtained through certain generalization of the concept of cohomology groups. From the thus generalized standpoint the ordinary cohomology theory of finite groups is seen as a local theory at a point in a certain space. Besides the interest of the thus obtained so to say global cohomology theory itself, this generalization is effective in applications of the cohomology theory. The author will discuss it before long in other chance.

1. Let G be a finite group, A be an abelien group such that G induces some automorphisms in A as a right operator group, i.e. for each s in G

$$a \to a^s \qquad (a \in A,\ s \in G)$$

is an automorphism and

$$(a^s)^t = a^{st} \qquad (t \in G).$$

Let Z be the ring of rational integers, $||$ be a normalized valuation of the rational number field R. We denote by $Z_{||}$ the ring Z of rational integers itself or the ring of l-adic integers, according to each of cases when $||$ is the normalized archimedien valuation $||_\infty$ or when $||$ is a normalized non archimedien valuation $||_l$ corresponded to a prime natural number l, respectively. Let D be a representation of G with regular matrices with coefficients in $Z_{||}$. We call *such a pair* of a normalized valuation of R and a representation of G in $Z_{||}$ as a point in the space of cohomology of G. From now on we define the local cohomology group of G with A as coefficients at a point in the space of cohomology of G as follows.

2. Let $A^{||i}$ for $i=1,2,3,\dots$ be the trivial subgroup in A consisting only of the unit element e or the subgroup consisting of every l^i-th power of elements in A, according to each of cases when $||=||_\infty$ or when $||=||_l$, respectively. Let $A_i^{(||)}$ denote the quotient group $A/A^{||i}$. As $A^{||i}$ is G-subgroup, $A_i^{(||)}$ is a G-right group. Let $\bar{A}^{(||)}$ be the inverse limit group

$$\bar{A}^{(||)} = \varprojlim [A_i^{(||)};\ L_i^{i+j}] \quad (i=1,2,\dots;\ j=0,1,\dots)$$

where we denote by L_i^{i+j} the natural homomorphism of $A_{i+j}^{(||)}$ onto