117. *Counter Examples to Wallace's Problem*[1)]

By Naoki KIMURA and Takayuki TAMURA
Tokyo Institute of Technology and Gakugei Faculty of Tokushima University

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A. D. Wallace proposed in his paper[2)] the following problem:

If a compact mob[3)] *has a unique left unit, is this also a right unit?*

In this short note we shall show counter examples to the above-mentioned problem without proof. We will write elsewhere[4)] in these connections and about related topics with detailed discussion. Example 1 is given by N. Kimura and Example 2 by T. Tamura.

Example 1. Let S be a set of all pairs (x, y) such that $0 \leqq x \leqq y \leqq 1$. Consider S as a topological space with the usual 2-dimensional plane topology, as well as a multiplicative system with multiplication;

$$(x, y)(x', y') = (xx', xy'),$$

where the multiplication in the parentheses at the right hand side will be understood as usual one.

Then S becomes a compact connected Hausdorff semigroup, and $(1, 1)$ is a unique left unit. Moreover S has no right unit.

Example 2. Let A be a compact connected mob with two-sided unit 1 and two-sided zero 0. Such a mob A really exists, for example, the interval of real numbers from 0 to 1 with the usual topology and multiplication. Let us consider a compact connected Hausdorff space B. If A and B are given, we can construct the union S of A and B such that A has only one 0 in common with B by identifying abstractly one element of B with 0 in A. The product xy in S is defined as the following manner:

$$xy = \begin{cases} x \cdot y & \text{for } x, y \in A, \\ 0 & \text{for } x \in B,\ y \in A, \\ y & \text{for } x \in S,\ y \in B, \end{cases}$$

where $x \cdot y$ is the product of x and y in A. Next we shall introduce a topology into S. The neighborhood $N(x)$ of x is defined as the following manner:

if $0 \neq x \in A$, $N(x) = U(x)$ where $U(x)$ is a neighborhood of x in A,

if $0 \neq x \in B$, $N(x) = V(x)$ where $V(x)$ is a neighborhood of x in B,