166. *Simplification of the Canonical Spectral Representation of a Normal Operator in Hilbert Space and Its Applications*

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If we denote by $K(z)$ the complex resolution of the identity associated with a normal operator N [1] in the abstract Hilbert space $\mathfrak{H}$ which is linear, metric, complete, infinite-dimensional, and separable, and by G the entire complex z-plane, then $N=\int_G z dK(z)$, as is well known.

One of the aims of this note is to outline that if N is an invertible [3] normal operator whose point spectrum is not empty, the above spectral representation of N can be reduced to a line projection-integral, and the other is to outline its applications to few studies on the distribution of the spectrum and the resolvent set of N and to the problem of unitary equivalence of invertible normal operators whose point spectra are not empty. Details of these studies will be, however, shortly published in Memoirs of the Faculty of Education of Kumamoto University.

By the operational calculus we can first prove the following lemmas:

Lemma 1. Let U be a unitary operator in $\mathfrak{H}$; let α and β be arbitrary complex numbers such that $\alpha\overline{\beta}\neq\overline{\alpha}\beta$ and $\overline{\beta}/\beta$ belongs to the resolvent set of U, under the assumption that there exist complex numbers with absolute value 1 in the resolvent set of U; and let H be the operator defined by the relation $H(\beta U-\overline{\beta}I)=\alpha U-\overline{\alpha}I$. Then U and H are permutable, and $(\alpha l-\overline{\alpha})(\beta l-\overline{\beta})^{-1}$ belongs to the point spectrum, to the continuous spectrum, or to the resolvent set of H, according as $l(\neq\overline{\beta}/\beta)$ belongs to the point spectrum, to the continuous spectrum, or to the resolvent set of U; and the converse is also valid. Moreover the characteristic projection of U for a characteristic value l is identical with that of H for the corresponding characteristic value $(\alpha l-\overline{\alpha})(\beta l-\overline{\beta})^{-1}$.

Lemma 2. The preceding lemma holds also in the case where $\overline{\beta}/\beta$ belongs to the continuous spectrum of U.

Remark. In Lemma 1 H is a bounded self-adjoint operator,