51. On Compact Semi-groups

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In this note, we shall extend a theorem of my paper [4], and apply the theorem to study the structure of compact semi-groups.[1]

A semi-group S of elements $a, b, c \cdots$ is called a *homogroup* if

1) S contains an idempotent e,
2) for each a of S, there exist elements a', a'' such that

$$aa' = e = a''a,$$

3) for every a of S,

$$ae = ea.$$

The terminology "homogroup" has been used by G. Thierrin [8], A. H. Clifford and D. D. Miller [1] have used the "semi-group having zeroid elements". In my paper [4], we proved the following

Theorem 1. Any compact commutative semi-group is homogroup.

The similar theorem has been also obtained by R. J. Koch [6].

Definition. A semi-group S is called *reversible* (following G. Thierrin [9]), if for any two elements a and b, $aS \frown bS \neq 0 \neq Sa \frown Sb$.

It is easily seen that *any commutative semi-group* or *any semi-group with zero-element is reversible*. We shall prove Theorem 2 which is a generalisation of Theorem 1.

Theorem 2. A compact semi-group is homogroup, if and only if it is reversible.

Such a theorem for finite semi-group has been proved by G. Thierrin [9].

Proof. Suppose that S is compact homogroup, then S contains an idempotent e, and, for any two elements a and b, there are two elements a', b' such that

$$aa' = e = bb' \qquad a', b' \in S.$$

Therefore $aS \frown bS \ni e$. Similarly $Sa \frown Sb \ni e$. This shows that $aS \frown bS$, $Sa \frown Sb$ are non-empty.

Conversely, suppose that S is reversible, if S contains zero-element 0, Theorem 2 is clear. Suppose that S does not contain zero-element 0. By the compactness of S, S contains at least one closed right minimal ideal A (for detail, see K. Iséki [4]). Suppose that B is a closed minimal right ideal of S different from A. Let

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1) For general theory of semi-groups, see P. Dubreil [3].