74. *Contributions to the Theory of Semi-groups.* III

By Kiyoshi ISÉKI
Kobe University

(Comm. by K. KUNUGI, M.J.A., May 15, 1956)

Recently, the present author [2, 3] and Miss Y. Miyanaga [4] showed that some well-known theorems on compact abelian semi-groups were generalized to homogroups.

Our purpose is to generalize a theorem on compact abelian semi-groups to arbitrary homogroups.

A semi-group S is called a *homogroup,* if

1) S contains an idempotent e,

2) for every element x of S, there are two elements x' and x'' such that $xx'=e=x''x$,

3) for every element x of S, $xe=ex$.

Some writers have proved that any simple compact abelian semi-group is a compact group. We shall prove the following

Theorem 1. Any simple homogroup is a group.

For a semi-group with topology, we have

Theorem 2. Any simple compact homogroup is a compact group.

As any compact abelian semi-group is homogroup (see K. Iséki [1]), by Theorem 1, we have the following

Theorem 3. Any simple compact abelian semi-group is compact group.

The proof of Theorem 1. To prove it, we shall use the following lemma on simple semi-groups by D. Rees [5].

Lemma. A necessary and sufficient condition for semi-group S to be simple is that $SxS=S$ for every element x of S.

Let S be a simple homogroup, and e the idempotent stated in the condition 1), then, by the simplicity of S and Lemma, we have

$$SeS=S.$$

By the condition 3), we have

$$(*) \qquad S\cdot Se=S.$$

$S^2(=S\cdot S)$ is a two-sided ideal of S and contains the idempotent e. If $S^2=e$, then, by $(*)$, we have

$$e^2=e=S.$$

Hence S is a group. On the other hand, if $S^2=S$, $(*)$ implies

$$Se=S.$$

By a well-known theorem (see G. Thierrin [6]), the set Se is a group ideal of S. Therefore S is a group. The proof is complete.