163. On Interpolations of Analytic Functions. II
(Fundamental Results)

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2. In this Note we consider a generalization of the result mentioned in the introduction of this paper.

Let D be a bounded closed points set whose complement K with respect to the extended plane is connected and regular in the sense that K possesses a Green's function with pole at infinity. Let $w=\phi(z)$ map K onto the region $|w|>1$ so that the points at infinity correspond to each other. Let Γ_ρ be the level curve determined by $|w|=\rho>1$.

Let the sequence of points (P) which lie on D satisfy the condition that the sequence of functions

$$\frac{W_n(z)}{\Delta^n w^n} = \frac{(z-z_1^{(n)})(z-z_2^{(n)})\cdots(z-z_n^{(n)})}{[\Delta\phi(z)]^n}$$

converges to a function $\lambda(w)$, single valued, analytic and non-vanishing for w exterior to the unit circle $|w|=1$, and uniformly on any bounded closed points set exterior to the unit circle, that is

$$(17) \qquad \lim_{n\to\infty}\frac{W_n(z)}{[\Delta w]^n} = \lambda(w) \neq 0 \qquad \text{for} \qquad |w|>1,$$

where Δ is the capacity of D.

Let $f(z)$ be a function single valued and analytic throughout the interior of the level curve $\Gamma_\rho : |w|=|\phi(z)|=\rho>1$ but not analytic regular on Γ_ρ. Then the sequence of polynomials $P_n(z; f)$ of respective degrees n which interpolate to $f(z)$ in all the zeros of $W_{n+1}(z)$ is given by

$$(18) \qquad P_n(z; f)=\frac{1}{2\pi i}\int_{\Gamma_R}\frac{W_{n+1}(t)-W_{n+1}(z)}{W_{n+1}(t)}\frac{f(t)}{t-z}dt: \qquad (1<R<\rho)$$

and we have, for z which satisfies $|\phi(z)|=|w|<R$,

$$(19) \quad R_n(z; f)\equiv f(z)-P_n(z; f)=\frac{1}{2\pi i}\int_{\Gamma_R}\frac{W_{n+1}(z)}{W_{n+1}(t)}\frac{f(t)}{t-z}dt: \quad (1<R<\rho).$$

In this case we have the following theorem.

Theorem 1. *Let D be a closed limited points set whose complement K with respect to the extended plane is connected and regular in the sense that K possesses a Green's function with pole at infinity. Let $W=\phi(z)$ map K onto the region $|w|>1$ so that the points at infinity correspond to each other.*

Let the function $f(z)$ be single valued and analytic throughout