162. On Interpolations of Analytic Functions. I

(Preliminaries)

By Tetsujiro KAKEHASHI

(Comm. by K. KUNUGI, M.J.A., Dec. 13, 1956)

Walsh[1] has proved the following theorem: *Let $f(z)$ be a function single valued and analytic within the circle $C_\rho: |z|=\rho>1$, but not analytic regular on C_ρ. Then the sequence of polynomials $P_n(z;f)$ of respective degrees n found by interpolation to $f(z)$ in all the zeros of polynomials $Z^{n+1}-1$ converges to $f(z)$ throughout the interior of the circle C_ρ, uniformly on any closed set interior to C_ρ and diverges at every points exterior to C_ρ as n tends to infinity.* He has mentioned the possibility of a generalization of this theorem in his paper.

For the convergence of sequences of polynomials found by interpolations in sets of points which satisfy a certain condition, a complete result has been shown by Walsh,[2] but for the divergence, problems have been left unsolved.

For this divergence problem of such a sequence, a few works have been done by the author,[3)-5)] but these results were not satisfactory. But soon afterwards a little satisfactory result has been obtained by the author:[6]

Let the sequence of points

$$(P)\qquad \begin{cases} z_1^{(1)} \\ z_1^{(2)}, z_2^{(2)} \\ z_1^{(3)}, z_2^{(3)}, z_3^{(3)} \\ \cdots\cdots\cdots \\ z_1^{(n)}, z_2^{(n)}, z_3^{(n)}, \cdots, z_n^{(n)} \\ \cdots\cdots\cdots \end{cases}$$

which do not lie exterior to the unit circle $C: |z|=1$, satisfy the condition that the sequence of

1) J. L. Walsh: The divergence of sequences of polynomials interpolating in roots of unity, Bulletin Am. Math. Soc., **12**, 715 (1936).

2) J. L. Walsh: Interpolation and approximation, Am. Math. Soc. Coll. Publ., **20** (1935).

3) T. Kakehashi: On the convergence-region of interpolation polynomials, Jour. Math. Soc. Japan, **7**, 32 (1955).

4) T. Kakehashi: The divergence of interpolations. I–III, Proc. Japan Acad., **30**, Nos. 8,9,10 (1954).

5) T. Kakehashi: Integrations on the circle of convergence and the divergence of interpolations. I, Proc. Japan Acad., **31**, No. 6, 329 (1955).

6) T. Kakehashi: The decomposition of coefficients of power-series and the divergence of interpolation polynomials, Proc. Japan Acad., **31**, No. 8, 517 (1955).