8. *The Initial Value Problem for Linear Partial Differential Equations with Variable Coefficients. I*

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Recently Yosida[1] gives the existence theorem for Cauchy problem of the wave equation by semigroup-like method, but it seems me not so adequate to obtain solutions for more general equations such that their coefficients depend on the time.

The object of this paper is to give an existence theorem for Cauchy problem in the whole space, and gives some generalization of Leray's results.[2] Our main idea owes to Yosida.

1. Notations. We consider only a real function or a smooth system of real linear differential operator defined over l-Euclidean vector space R_x^l and $l+1$-Euclidean vector space $R_t^1\times R_x^l$: that is, their coefficients have bounded derivatives of all orders.

Let $B\left(x, \frac{\partial}{\partial x}\right)=\left(b_{ij}\left(x, \frac{\partial}{\partial x}\right)\right)$ $(i, j=1,2,\cdots, m)$ be an (m, m)-smooth system of differential operator defined over R_x^l. Let $s=\{s(i)\mid i=1,2, \cdots, m\}$ be the set of non negative integers such that the order $o(b_{ij})$ of b_{ij} does not exceed $s(i)+s(j)$ and denote by $b'_{ij}\left(x, \frac{\partial}{\partial x}\right)$ the sum of terms in b_{ij} with order exactly $s(i)+s(j)$. Then we call that $B\left(x, \frac{\partial}{\partial x}\right)$ is *uniformly strongly elliptic*, if there is a positive ρ such that

$$b'_{ij}(x, i\xi)\, v_i\bar{v}_j \geqq \rho\sum_{i=1}^{m}|\xi|^{2s(i)}\,|v_i|^2$$

for all real scalars $\xi:(\xi_1,\cdots,\xi_l)$, all $x\in R_x^l$ and all complexes $v_1,\cdots,v_m$. By the smoothness of $B\left(x, \frac{\partial}{\partial x}\right)$ it is equivalent to the following: $(b'_{ij}(x, i\xi))$ is positive definite for all $x\in R_x^l$ and all real scalars $\xi:|\xi|=1$. Then from Leray's method, it is shown that there are positive k, α and β such that

$$((u, v))_B=\int_{R_x^l}\left(\left(B\left(x, \frac{\partial}{\partial x}\right)+B\left(x, \frac{\partial}{\partial x}\right)^* +k\right)u, v\right)dx,$$

1) Cf. K. Yosida: An operator-theoretical integration of the temporally inhomogeneous wave equation (to appear).

2) Cf. L. Leray: Hyperbolic equations with variable coefficients, Princeton, N. J. (1954).