35. On a Right Inverse Mapping of a Simplicial Mapping

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1. Let X and Y be topological spaces and let f be a continuous mapping from X onto Y. By a right inverse mapping of f, we mean a continuous mapping g of Y into X such that $fg(y)=y$ for each point y of Y. In the present note, we shall show that, in case X and Y are (finite or infinite) simplicial complexes and f is a simplicial mapping from X onto Y, the existence of a right inverse mapping of f is equivalent to some combinatorial properties of X and Y. The theorem will be stated in 3. In 2 we shall state notations and a lemma which we need later on.

2. We denote by J the additive group of integers. By a *lower sequence* of abelian groups, we mean sequences of abelian groups $\{G_i; i\in J\}$ and homomorphisms $\{g_i; i\in J\}$ such that

i) g_i is a homomorphism of G_{i+1} into G_i, $i\in J$;

ii) $g_i g_{i+1}$ is the zero-homomorphism, $i\in J$.

By a *homomorphism* of a lower sequence $\{G_i; g_i\}$ of abelian groups into a lower sequence $\{H_i; h_i\}$ of abelian groups, we mean a sequence $\{f_i; i\in J\}$ of homomorphisms such that

i) f_i is a homomorphism of G_i into H_i, $i\in J$;

ii) $h_i f_{i+1}=f_i g_i$, $i\in J$.

A homomorphism $\{f_i\}$ of a lower sequence $\{G_i; g_i\}$ into a lower sequence $\{H_i; h_i\}$ is called a *retraction-homomorphism* if and only if there exists a homomorphism $\{k_i\}$ of $\{H_i; h_i\}$ into $\{G_i; g_i\}$ such that, for each integer $i\in J$, $f_i k_i$ is the identity isomorphism of H_i into H_i.

Let X be a simplicial complex. We denote the i-section of X by X^i. Let A be a subcomplex of X. By the barycentric subdivision of X relative to A, we mean the barycentric subdivision of X such that all simplexes of A are not subdivided (cf. [1] or [3]).

Lemma. *Let X and Y be simplicial complexes and let f be a simplicial mapping of X into Y. Let B be a subcomplex of Y. Let us denote the first barycentric subdivisions of X and Y relative to the subcomplexes $f^{-1}(B)$ and B by $\tilde{X}$ and $\tilde{Y}$, respectively. Then there exists a simplicial mapping $\tilde{f}$ of $\tilde{X}$ into $\tilde{Y}$, which we call a simplicial mapping associated with f and B with the following property: Let s and s' be simplexes of $X-f^{-1}(B)$ and $Y-B$. Then we have $f(s)=s'$ if and only if the barycenter of s is mapped into the barycenter of s' by $\tilde{f}$.*