46. *Some Operations on the Ranked Spaces. I*

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Prof. K. Kunugi introduced the notion of the ranked spaces in the Note "Sur les espaces complets et régulièrement complets. I".[1] It is the purpose of this Note to study the product spaces of ranked spaces and the function spaces $F(E,G)$ which denotes the totality of functions of a fixed set E into a fixed ranked space G.[2]

1. In [I] it is required that the system of neighbourhoods satisfies F. Hausdorff's axiom (C).[3] We shall attempt to exclude this hypothesis.

Let R be a space whose topology is given by a system of neighbourhoods which satisfies F. Hausdorff's axiom (A).[3] Then we can calculate the depth[4] of R and introduce the notion of the ranked spaces according to [I]. We shall conform ourselves, without contrary indication, to the notions and the terminology of [I]. But it is necessary to modify some notions as follows.

Definition 1. A ranked space is called to be *complete*[6] if, for every fundamental sequence $v_\alpha(p_\alpha)$, $0\leqq\alpha\leqq\omega_\mu$, the following conditions (1), (2) are satisfied:

(1) $$\bigcap_\alpha v_\alpha(p_\alpha)\neq 0.$$

(2) $$\bigcap_\alpha I\{v_\alpha(p_\alpha)\}^{5)}\neq 0 \quad \text{if } \omega_\mu<\omega_\nu.$$

Definition 2. A set E is called to be *non-dense*[7] if, for every point p of R and every neighbourhood $v(p)$ of p, $I\{v(p)\}\nsubseteq\bar{A}$. A set F is called to be of the first category if it is a union of an ω_ν-sequence of non-dense sets: $F=\bigcup_{0\leq\alpha<\omega_\nu}F_\alpha$ where every F_α is non-dense.

Then we can prove Baire's theorem:

Theorem 1. *In the complete ranked spaces any non-empty open set is not of the first category.*

Proof. For proving the theorem it is sufficient to show that, if G is a non-empty open set and $E_{2\alpha}$, $0\leq\alpha<\omega_\nu$, are non-dense sets, then $G\neq\bigcup_\alpha E_{2\alpha}$.

Since E_0 is non-dense, $G\nsubseteq\bar{E}_0$. Therefore there exist a point p_0, a rank γ_0 and $v_0(p_0)$ of rank γ_0 such that $v_0(p_0)\subseteq G$ and $v_0(p_0)\cap E_0=0$. Suppose that we have already defined p_β,γ_β and $v_\beta(p_\beta)$ for all β such that $0\leq\beta<\alpha$ where $0<\alpha<\omega_\nu$ and they satisfy the following conditions (3) and (4):

(3) $$v_0(p_0)\supseteqq v_1(p_1)\supseteqq\cdots,\ \gamma_0\leq\gamma_1\leq\cdots,\ v_\beta(p_\beta)\in\mathfrak{V}_{\gamma_\beta}.$$