74. On the Divisibility of Dedekind's Zeta-Functions

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Let k be an algebraic number field of finite degree and K a finite extension over k. Then it was conjectured by E. Artin [2] that the Dedekind's zeta-function $\zeta_k(s)$ of k divides the Dedekind's zeta-function $\zeta_K(s)$ of K, in the sense that the quotient $\zeta_K(s)/\zeta_k(s)$ is an integral function of the complex variable s. Already R. Dedekind [4] has proved that $\zeta_k(s)$ divides $\zeta_K(s)$, if K is a "rein" cubic extension of the rational field k. E. Artin [2], H. Aramata [1] and R. Brauer [3] have made contributions to this conjecture and obtained indeed affirmative answers in several special cases.

In this paper, using Artin's L-series and Brauer's group-theoretical lemma, I shall prove a theorem which includes all those former results as special cases. And here I wish to express my hearty gratitude to Prof. Z. Suetuna for his encouragement.

In the following, for sake of simplicity, we shall use the following notation: If U is a finite group, θ the character of the regular representation of U and λ_0 the principal character of U, then we shall denote the character $\theta-\lambda_0$ by $X(U)$.

Lemma 1 (R. Brauer [3]). Let G be a group of finite order g. Then the character $X(G)$ of G can be expressed as follows:

$$X(G)=\sum_{H_\sigma}\sum_j c_j^{(\sigma)}\,\Xi_{\psi_j^{(\sigma)}}, \tag{1}$$

where H_σ ranges over all the cyclic subgroups of order $h_\sigma>1$ of G and $\Xi_{\psi_j^{(\sigma)}}$ over the characters of G induced by all the irreducible characters $\psi_j^{(\sigma)}$ of H_σ. Furthermore, the coefficients $c_j^{(\sigma)}$ of $\Xi_{\psi_j^{(\sigma)}}$ in (1) are non-negative rational numbers with denominators g, and given by

$$c_j^{(\sigma)}=\frac{1}{g}\{\varphi(h_\sigma)-\sum_{\sigma^*}\psi_j^{(\sigma)}(\sigma^*)\}, \tag{2}$$

where σ^* ranges over all the generators of H_σ.

Remarking that the numerator of $c_j^{(\sigma)}$ depends only on H_σ and $\psi_j^{(\sigma)}$, we have the following important

Lemma 2. Let G and g be the same as in Lemma 1. Let H be an arbitrary subgroup of order $h>1$ of G. Then we can rewrite (1) as follows:

$$X(G)=\frac{h}{g}\Xi_{X(H)}+\sum_{H_{\sigma'}\not\subseteq H}'\sum_j c_j^{(\sigma')}\,\Xi_{\psi_j^{(\sigma')}}, \tag{3}$$