# 88. On Dowker's Problem

By Yoshiaki HAYASHI

(Comm. by K. KUNUGI, M.J.A., July 12, 1957)

In 1949, C. H. Dowker raised the question[1] 'Is every normal Hausdorff space $R$ countably paracompact (i.e. does every countable open covering of $R$ have a locally finite open refinement)?'. In this paper, we shall give a negative answer to this problem, i.e. we shall show that there exists a normal Hausdorff space which is not countably paracompact.

(1) Let

$R_1=\{0, 1, 2, 3, \cdots, \omega, \cdots, \Omega\}$ where $\Omega$ is the first ordinal number in all 3rd-class ordinals,

$R_2=R_3=\cdots=R_n=\cdots=\{0, 1, 2, 3, \cdots, \omega\}$ where each $n<\infty$, $\omega$ is the first ordinal in all 2nd-class ordinals.

For each $R_i$, we define its topology by the limit of ordinals as usual.[2]

Let

$$S=R_1\times R_2\times R_3\times\cdots .$$

Give the weak topology of the product space for $S$.

Since each $R_i$ is compact Hausdorff space, $S$ is a compact Hausdorff space. And, therefore $S$ is normal.

Now, $(\Omega, \omega, \omega, \omega, \cdots)$ is a point of $S$. Let

$$R=S-(\Omega, \omega, \omega, \omega, \cdots).$$

(2) Since $R$ is a subspace of $S$, $R$ is a Hausdorff space. We shall prove that $R$ is normal.

Let $A, B$ be disjoint two closed sets of $R$.

Let $\bar{A}$ be the closure in $S$ of $A$, and $\bar{B}$ be the closure in $S$ of $B$.

(i) The case of $\bar{A}\frown\bar{B}\not\ni(\Omega, \omega, \omega, \cdots)$.

$\bar{A}, \bar{B}$ are disjoint two closed sets of $S$. Since $S$ is normal, there exist disjoint two open sets $G_0, H_0$ of $S$ such that $G_0\supset\bar{A}$, $H_0\supset\bar{B}$. $G=R\frown G_0$, $H=R\frown H_0$ are disjoint two open sets of $R$ such that $G\supset A$, $H\supset B$.

(ii) The case of $\bar{A}\frown\bar{B}\ni(\Omega, \omega, \omega, \cdots)$.

This case never happen.

Assume $\bar{A}\frown\bar{B}\ni(\Omega, \omega, \omega, \cdots)$.

1) See [1]. (Numbers in brackets refer to the references at the end of the paper.)

2) We define neighbourhoods of $p$ as follows; for each $q<p$, $\{p'\mid q<p'\leqq p\}$ is a neighbourhood of $p$.