153. Note on Idempotent Semigroups. I

By Naoki KIMURA
Tokyo Institute of Technology and Tulane University of Louisiana
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§ 1. The purpose of this paper is to present the structure theorem of some special idempotent semigroups, called regular (see definition below), which would be considered to form a fairly wide category of idempotent semigroups.

A semigroup is called *left singular* (*right singular, rectangular*) if it satisfies an identity $ab=a$ ($ba=a$, $aba=a$). All of them are idempotent. Also a left (right) singular semigroup is rectangular. Further, a rectangular semigroup is the direct product of a left singular semigroup and a right singular semigroup. This direct product decomposition is unique up to isomorphism.

A global structure theorem on idempotent semigroup was carried out by David McLean [1]. The theorem can be stated as follows:

Let S be an idempotent semigroup. Then there exist, up to isomorphism, a unique semilattice Γ, and a disjoint family of rectangular subsemigroups of S indexed by Γ, $\{S_\gamma : \gamma \in \Gamma\}$, such that

(i) $S=\bigcup\{S_\gamma : \gamma \in \Gamma\}$,

and (ii) $S_\alpha S_\beta \subset S_{\alpha\beta}$ *for all* $\alpha, \beta \in \Gamma$.

In what follows we call Γ the *structure semilattice* of S, and S_γ the *γ-kernel*. And we denote the decomposition by the notation

$$S \sim \sum\{S_\gamma : \gamma \in \Gamma\},$$

and call it the *structure decomposition* of S.

REMARK. It is to be noted that there are, in general, non-isomorphic idempotent semigroups which have isomorphic structure semilattices and the corresponding isomorphic kernels.

§ 2. Now we shall settle here the necessary and sufficient condition for an idempotent semigroup whose every kernel is left (right) singular.

Before going into the theorem, we need the following definitions.

An idempotent semigroup S is called (1) *left regular*, (2) *right regular*, (3) *regular*, if it satisfies the following corresponding identities:

(1) $aba=ab$,

(2) $aba=ba$,

(3) $abaca=abca$.

REMARK. We can replace idempotency of S by the weaker condition $S^2=S$ in the definition of left (right) regularity above.