148. On Non-linear Partial Differential Equations of Parabolic Types. III

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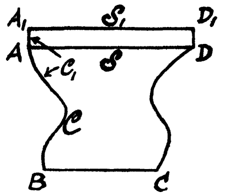
Here we give the main existence theorem and discuss on the regularity of domains. We also extend the meaning of the heat operator in the higher dimensional space in the final section.

8. Existence theorem (II). THEOREM 8.1. *Suppose that $f(x, y, u)$ is continuous, quasi-bounded with respect to u and satisfies the condition (Lk) on $(C, S]\times(-\infty, \infty)$ and the condition (P) is satisfied for the equation (E_1) and the bounded function $\beta(x, y)$ given on C. Then there exists a continuous solution of (E_1) on $(C, S]$.*

PROOF. Since $(C, S]$ is a p-domain, the end points A and D of the segment S also form end points of the curve C. Prolonging the curve C upward from A and D by length δ, we get the points A_1 and D_1. Denote the segment A_1D_1 by S_1 (not including its end points), and denote by C_1 the curve which consists of the segment A_1A, the curve C and the segment D_1D. Then $(C_1, S_1]$ is also a p-domain.



Now we extend the function $f(x, y, u)$ to $(C_1, S_1]$ as follows: if (x, y) belongs to the interior of the rectangle A_1ADD_1 or on the segment S_1 we put $f(x, y, u)=f(x, b, u)$ where b is the y-coordinate of A or D. Since there is no ambiguity, we permit ourselves to write $f(x, y, u)$ for the extended function. The function $\beta(x, y)$ given on C can be extended from C to C_1 in the same way, i.e. if (x, y) belongs to A_1A or D_1D then we put $\beta(x, y)=\beta(x, b)$. We write also $\beta(x, y)$ for the extended function.

Next we extend the Ψ_β-function $\psi(x, y)$ on $[C, S]$ to the Ψ_β-function on $[C_1, S_1]$ as follows: on the rectangle A_1ADD_1 $\psi(x, y)$ is equal to a continuous solution of (E_1) with the boundary value $\psi(x, y)$ on the closed segment AD, $\psi(A)$ on A_1A and $\psi(D)$ on D_1D. This continuous solution does exist. Indeed, to find such a solution we shall first solve the equation of heat conduction with the given boundary condition, and let $h(x, y)$ be a solution of it. We shall consider the equation $\Box v=f(x, y, v+h(x, y))$. Since $f(x, y, u)$ is quasi-bounded with respect to u on $(C, S]$, $f(x, y, v+h(x, y))$ is also quasi-bounded with respece to v on $(C_1, S_1]$, therefore by Theorem 4.2 there is a solution $v(x, y)$ satisfying $\Box v=f(x, y, v+h(x, y))$ and vanishing on the boundary.