76. *On Tangent Bundles of Order 2 and Affine Connections*

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(Comm. by Z. SUETUNA, M.J.A., June 12, 1958)

In this paper, the author will show that the classical connections, for instance, the affine, projective, conformal connections, can be considered from a unificative standpoint by means of the concept of tangent bundles of order 2, although they can be also discussed through the theory of connections of vector bundles.[1] We shall investigate the relations between this theory and the ones of C. Ehresmann and S. S. Chern[2] in Mathematical Journal of Okayama University, 8.

1. **The group $\mathfrak{L}_n^2$.** According to C. Ehresmann,[3] let L_n^2 be the group of the infinitesimal isotropies of order 2 at the origin of R^n, whose any element is written as a set of numbers (a_i^j, a_{ik}^j) such that $|a_i^j| \neq 0$, $a_{ik}^j = a_{ki}^j$. We can easily see that the set $\mathfrak{L}_n^2$ of (a_i^j, a_{ik}^j) such that only $|a_i^j| \neq 0$, also forms a group containing L_n^2 as a subgroup with the multiplication as follows:

For any two $\alpha, \beta \in \mathfrak{L}_n^2$, $\gamma = \alpha\beta$ is defined by

$$a_i^j(\gamma) = a_k^j(\alpha)\, a_i^k(\beta), \tag{1.1}$$

$$a_{ik}^j(\gamma) = a_h^j(\alpha)\, a_{ik}^h(\beta) + a_{hl}^j(\alpha)\, a_i^h(\beta)\, a_k^l(\beta). \tag{1.2}$$

By (1.1), we have a natural homomorphism σ: $\mathfrak{L}_n^2 \to L_n^1 = GL(n, R)$ such that

$$a_i^j(\sigma(\alpha)) = a_i^j(\alpha). \tag{1.3}$$

As is well known, we may consider L_n^1 as a subgroup of L_n^2, regarding the second coordinates a_{ik}^j of their elements as zero. Let $\mathfrak{N}_n^2$ be the kernel of σ. By means of (1.2), for any $\alpha, \beta \in \mathfrak{N}_n^2$, we have

$$a_{ik}^j(\alpha\beta) = a_{ik}^j(\alpha) + a_{ik}^j(\beta),$$

hence $\mathfrak{N}_n^2$ is a vector space of dimension n^3. We define a mapping $\eta: \mathfrak{L}_n^2 \to \mathfrak{N}_n^2$ by

$$\eta(\alpha) = \sigma(\alpha^{-1})\alpha. \tag{1.4}$$

Then, we can write uniquely any element α of $\mathfrak{L}_n^2$ as a product of $\sigma(\alpha) \in L_n^1$ and $\eta(\alpha) \in \mathfrak{N}_n^2$

1) See T. Ōtsuki: Geometries of Connections (in Japanese), Kyoritsu Shuppan Co. (1957).

2) C. Ehresmann: Les connexions infinitésimales dans un espace fibré différentiable, Colloque de Topologie (Espaces fibrés), 29–55 (1950); S. S. Chern: Lecture note on differential geometry, Chicago University (1950).

3) See C. Ehresmann: Les prolongements d'une variété differentiable I. Calcul des jets, prolongement principal, C. R. Acad. Sci., Paris, **233**, 598–600 (1951).