

75. Hukuhara's Problem for Hyperbolic Equations with Two Independent Variables. I. Semi-linear Case

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Introduction. In the Cauchy problem for partial differential equations involving N unknown functions u_i ($i=1, \dots, N$) of two independent variables, the initial values of the functions u_i are prescribed on a curve. Generalizing the way of giving the Cauchy conditions, we can set up another problem where N curves C_i are given and on each C_i the value of u_i is prescribed. This will be called *Hukuhara's problem*, or shortly, Problem H, since M. Hukuhara has originally studied an analogous problem for the system of ordinary differential equations.

Generally speaking, the solutions of the Cauchy problems for hyperbolic systems are *stable*, that is to say, the *slight* change of the initial values entails but a *slight* change of the solutions. In physical problems, the initial values of unknown quantities are found experimentally and can not be determined with absolute precision, so that the stability property of the problem has a special importance in this case. However, the errors of the measurement may be committed also *concerning the situation of the initial curves*, i.e. the values of the different unknown quantities can not be measured at the exactly same time and place. This consideration leads us to the concept of the Hukuhara's problem, and the influence of the change of the situation of the curves C_i upon the solutions should be investigated to secure the stability property of the problem.

We shall show in this paper, the *correctly posedness* of the Hukuhara's problem for semi-linear hyperbolic systems with two independent variables, which are assumed to be of diagonal form, since every semi-linear system can be transformed to this normal form by the well-known techniques. Same results for the quasi-linear systems will be stated in the following report entitled Part II. Our proof is given by means of the elementary method of successive approximation, since the stability of the solutions concerning the situation of the curves C_i , one of the characteristic properties of our problem, can be obtained by it very easily.

Recently, analogous problems have been treated by two Polish mathematicians, Z. Szymdyt [2-5] and A. Pliś [1]. The former proved the existence and the uniqueness of solutions of a problem intimately