96. On Tangent Structures of Lower Dimensional Manifolds

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1. In this note we try to extend the Wu's result[1] about the homotopy classification of mappings of a certain 4-complex into the Grassmann manifold $\hat{R}_{n,4}$. In virtue of the topological invariance of the Pontrjagin class of a 4-manifold, the Wu's result implies that tangent structures of a compact orientable 4-manifold are independent of its differentiable structures. As is easily seen, any compact n-manifold ($n \leqq 3$) has such property.

2. Now we consider the cases $n \geqq 5$.

Proposition 1. *Suppose that the 4-dimensional cohomology group $H^4(M^5, Z)$ of a compact orientable 5-manifold M^5 has no 2-torsion, and that its 4-dimensional Stiefel-Whitney class W^4 is not zero. Then its tangent structures are equivalent if and only if their 4-dimensional Pontrjagin classes are coincident.*

Proposition 2. *Let $6 \leqq m \leqq 8$. Suppose that the 4-dimensional cohomology groups $H^4(M^m, Z)$ of m-manifolds M^m have no 2-torsions. Then their tangent structures are equivalent if and only if their 4-dimensional Pontrjagin classes are coincident.*

We do not know whether these restrictions imposed on the cohomology groups of the manifolds are superfluous or not. These propositions are proved with the standard procedure of homotopy extension by using several results[2] on the homotopy properties of Grassmann manifolds.

1) W.-T. Wu: Sur les classes caractéristiques des structures fibrées sphériques, Act. Sci. Ind., **1183**, 1-89 (1952), Proposition 7″.

2) W.-T. Wu: Les i-carrés dans une variété grassmannienne, C. R. Acad. Sci., Paris, **230**, 508-511 (1950). H. Toda, Y. Saito, and I. Yokota: Note on the generator of $\pi_7(SO(n))$, Mem. Coll. Sci. Univ. Kyôto, series A, **30**, 227-230 (1957).