95. *Some Expectations in AW*-algebras*

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1. Let A be a commutative AW^*-algebra (cf. [2]). We denote by B and P the totality of self-adjoint elements and projections in A, respectively. It is well known that A is isometrically isomorphic to the space $C(S)$ of all complex-valued continuous functions on a Stonean space S. In this representation, B (or P) is the totality of real-valued (or characteristic) functions in $C(S)$ which forms a conditionally complete vector lattice (or complete lattice) by the usual ordering in $C(S)$.

Let M be a left module over B. We shall call a mapping n of M into B an *n-mapping* on M if n satisfies

(1) $\quad n(x+y)\leq n(x)+n(y) \qquad (x, y\in M),$

(2) $\quad n(ax)=an(x) \qquad (x\in M, a\in A \text{ with } a\geq 0).$

If a mapping f of a subset $D(f)$ of M into B satisfies

(3) $\quad -n(-x)\leq f(x)\leq n(x),$

then we call f to be *n-bounded*. In the case when f is additive and when $D(f)$ is an additive subgroup of M, we can replace (3) by the inequality: $f(x)\leq n(x)$.

2. For convenience, we state a simple lemma which is easily verified.

Lemma 1. *Let M be a left module over (not necessarily commutative) AW^*-algebra L and $P(x)$ be a proposition concerning the element x in M. Suppose that the following two conditions are satisfied:*

(4) *If there exists a family $(e_i; i\in I)$ of orthogonal projections in L with l.u.b. 1 such that all $P(e_ix)$ are true, then $P(x)$ is true.*

(5) *For any projection e in L which is not zero, we can find a non-zero projection e' in L such that $e'\leq e$ and $P(e'x)$ is true. Then $P(x)$ is true.*

3. Now we state an extension theorem of Hahn-Banach type.

Theorem 1. *Let M be a left module over B with n-mapping n. Given an n-bounded B-module homomorphism of a B-submodule of M into B, it can be extended to an n-bounded B-module homomorphism of M into B.*

Proof. Let h be an n-bounded B-module homomorphism of a submodule $D(h)$ of M. Let R be the set of all couples $(f, D(f))$, where f is an n-bounded B-module homomorphism of a submodule $D(f)$ of M containing $D(h)$ into B such that $f=h$ on $D(h)$. If we define