# 92. An Investigation on the Logical Structure of Mathematics. XII[1)]

## The Principle of Extensionality and of Choice

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1. Principle of extensionality. The principle of extensionality means, roughly speaking, that a "set" is determined by its elements, or that two "sets" which have their elements in common are equal. On the other hand, if two "sets" are equal, they must have their properties in common. Accordingly, $a=b$ can be looked upon as the abbreviation of either one of the following formulas:

(1) $\forall x.x\in a\equiv x\in b,$

(2) $\forall x.a\in x\equiv b\in x.$

The former was adopted in UL[(I)], and therefore the formula

(I) $\forall xyz.x=y\wedge x\in z\Rightarrow y\in z$

expresses in UL the principle of extensionality. On the contrary, if we adopt the latter, the principle of extensionality is expressed by

(J) $\forall xyz.x=y\wedge z\in x\Rightarrow z\in y.$

Let, now, $P^c$ be the dependent variable in UL defined by

(3) $\forall u.u\in P^c\equiv c\in u.$

Namely, $P^c$ is the "set" of all properties of $c$. By using $P^c$ as a set[(X)] in a UL-proof, we shall prove that "(2) implies (1)" is a UL-theorem with (3) as unique premise. To prove this, we have only to give a UL-proof with (3) as premise and with

(4) $\forall x.a\in x\equiv b\in x.\wedge c\in a:\Rightarrow c\in b$

as conclusion, since it is clear that no contradiction can be deduced from (3).[2)] The UL-proof is simply as follows:

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1) For the present state of publication of this investigation, see footnote 0) in Part (IV), Nagoya Math. J., **13** (1958). The subtitles of Parts (I)-(XI) are as follows: (I) A logical system; (II) Transformation of proof; (III) Fundamental deductions; (IV) Compendium for deductions; (V) Contradictions of Russell's type; (VI) Consistent V-system; (VII) Set-theoretical contradictions; (VIII) Consistency of the natural number theory $T_1(N)$; (IX) Deduction of natural number theory in $T_1(N)$; (X) Concepts and sets; (XI) Underlying ideas of the investigation (in Japanese), The indices (I), (II), etc. attached in the following are the references to other Parts.

The contents of Parts (I)-(XI) were verbally published at the spring and autumn meetings, 1957, of the Mathematical Society of Japan and at the spring meeting, 1958, of the Japan Association for Philosophy of Science.

2) For $P^c$ is a T(0)- as well as T(V)-set[(VI)]. We can use, instead of $P^c$, the variable $Q^c$ defined by $\forall u.u\in Q^c\equiv c\notin u$ to prove (4).