91. *On Zeta-Functions and L-Series of Algebraic Varieties. II*

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Here I shall give some supplementary results to my previous paper [1].

Let k be a finite field with q elements. Then, for an abelian variety B defined over k, π_B denotes the endomorphism of B such that $\pi_B(b)=b^q$ for all points b on B and M_l denotes the l-adic representation of the ring of endomorphisms of B for a (fixed) rational prime l different from the characteristic of k.

1. Let A/V be a Galois (not necessarily unramified) covering defined over k, with group G and of degree n, where A is an abelian variety and V is a normal projective variety (both defined over k); let r be the dimensions of A and V. Then, in this section, we shall explain the behaviors of the zeta-function $Z(u, V)$ of V and the L-series $L(u, \chi, A/V)$ of A/V over k in the circle $|u|<q^{-(r-3/2)}$ and $|u|<q^{-(r-1)}$ respectively.

Now let η_σ be the automorphism of A induced by an element σ of G and let $\pi=\pi_A$. Then $Z(u, V)$ and $L(u, \chi, A/V)$ are given by the following logarithmic derivatives:

$$d/du\cdot\log Z(u, V)=\sum_{m=1}^{\infty}\{1/n\cdot\sum_{\sigma\in G}\det M_l(\pi^m-\eta_\sigma)\}u^{m-1},$$

$$d/du\cdot\log L(u, \chi, A/V)=\sum_{m=1}^{\infty}\{1/n\cdot\sum_{\sigma\in G}\det M_l(\pi^m-\eta_\sigma)\chi(\sigma)\}u^{m-1}.$$

First we shall calculate $\det M_l(\pi^m-\eta_\sigma)$. If we transform the representation M_l of G (i.e. the restriction of M_l to G such that $M_l(\sigma)=M_l(\eta_\sigma)$) into the following form:

$$(*)\qquad M_l\,|\,G=\begin{pmatrix} E_{d_1}\times 1 & & & 0 \\ & E_{d_\chi}\times F_\chi & & \\ & & E_{d_{\chi'}}\times F_{\chi'} & \\ 0 & & & \ddots \end{pmatrix},\ ^{1)}$$

where $1, F_\chi, F_{\chi'},\cdots$ are non-equivalent irreducible representations of G with characters $1, \chi, \chi',\cdots$ respectively, then, as $\pi\eta_\sigma=\eta_\sigma\pi$ for every σ in G, $M_l(\pi)$ must be transformed into the following form simultaneously:

$$(**)\qquad M_l(\pi)=\begin{pmatrix} (\pi_{ij}^{(1)})\times E_{f_1} & & & 0 \\ & (\pi_{ij}^{(\chi)})\times E_{f_\chi} & & \\ & & (\pi_{ij}^{(\chi')})\times E_{f_{\chi'}} & \\ 0 & & & \ddots \end{pmatrix},\ ^{1)}$$

where $(\pi_{ij}^{(\chi)})$ is a matrix of degree d_χ and f_χ is the degree of F_χ.

1) In the following, the matrices $E_{d_1}\times 1$ and $(\pi_{ij}^{(1)})\times E_{f_1}$ do not appear if $d_1=0$.